Low-lying nonrotational states in strongly deformed even–even nuclei of the rare-earth region

V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova
Joint Institute for Nuclear Research, Dubna

Fiz. Élem. Chastits At. Yadra **27**, 1643–1704 (November–December 1996)

The recently improved quasiparticle–phonon model of the nucleus is described. The calculated energies and wave functions of all the nonrotational states up to 2.3 MeV in ^{156,158,160}Gd, ^{160,162,164}Dy, and ^{166,168}Er are given, along with the probabilities for E1, E2, E3, E4, and M1 transitions from ground to excited states and the reduced probabilities of E1, E2, M1, and M2 transitions between excited states. The corresponding experimental data are systematized and compared with the results of calculations using the quasiparticle–phonon model. This model gives a fairly good description of the energies, the reduced probabilities of Eλ and Mλ transitions, and the largest two-quasiparticle configurations of one-phonon terms of the wave functions of nonrotational states. Some predictions are made. [S1063-7796(96)00306-3]

1. INTRODUCTION

Low-lying quadrupole and octupole collective and two-quasiparticle states in even–even deformed nuclei have been studied both experimentally and theoretically for almost half a century.[1–4] Nevertheless, the structure of the first vibrational states is still the subject of active discussion. For example, it is only a convention to associate the first $K_n^\pi = 0_1^+$ and 2_1^+ states, called beta- and gamma-vibrational states, with vibrations of the nuclear surface. Moreover, in many nuclei the first excited 0_1^+ state cannot be treated as a beta-vibrational state even by convention, owing to the low-intensity E2 transition to the ground-state rotational band. The existence of collective two-phonon (or doubly vibrational) states in strongly deformed nuclei remains uncertain. It is more difficult to find such states in deformed nuclei than in spherical nuclei because their energy centroid is shifted to higher excitation energies, where the level density is fairly large.[5,6]

Intensive experimental and theoretical studies have been carried out for low-lying magnetic and electric dipole excitations. The collective magnetic dipole excitation referred to as the scissors mode was discovered in (e,e') experiments in ^{156}Gd (Ref. 7) and was later found in almost all deformed nuclei. It was predicted in Ref. 8 using the two-rotor model. The collective properties of these excitations have been described in the random-phase approximation (RPA) in many studies.[9–12]

Interesting new information about the structure of excited states has been obtained by studying the probabilities of γ transitions between excited states. This information significantly augments that obtained from elastic and inelastic scattering, Coulomb excitation, one- and two-nucleon transfer reactions, and β decay.

The present review is a continuation of the first part in Ref. 13, where we described a version of the quasiparticle–phonon model (QPM) of the nucleus for even–even deformed nuclei. The QPM has been developed in conjunction with the study of the properties of deformed nuclei. The improvements of the QPM made after the writing of Ref. 13 are described in Sec. 2. The details of the calculations and the interaction constants are given in Sec. 3. The experimental data and results of the calculations are given in Sec. 4 in the form of two tables for each nucleus. The features of nonrotational excited states are discussed in Sec. 5, where we also compare theory and experiment. In Sec. 6 we formulate the main conclusions.

In this review we use the same notation as in Ref. 13 if not stated otherwise.

2. IMPROVEMENT OF THE QUASIPARTICLE–PHONON MODEL

The basic statements of the QPM are given in Refs. 14 and 15. The mathematical formalism of the QPM, designed to describe the energies and wave functions of low-lying nonrotational states in strongly deformed even–even nuclei, is described in Refs. 13 and 15.

As described earlier, the one-phonon states form a basis, which is used in the QPM as the basis of single-particle states. Special attention is paid to the construction of the phonon basis using the RPA. As is well known, the RPA is applicable to nuclei in which the ground-state correlations are small, i.e., when the number of quasiparticles $\langle \alpha_{q\sigma}^+ \alpha_{q\sigma} \rangle$ averaged over the ground-state wave function is small. The ground-state wave function of an even–even nucleus Ψ_0 is the phonon vacuum. According to the calculations of Ref. 16, the maximum number of quasiparticles in the ground states of ^{168}Er, ^{158}Gd, and ^{156}Gd is 0.017, 0.035, and 0.040, respectively. Since the number of quasiparticles in the ground states of strongly deformed nuclei is very small, the one-phonon states calculated in the RPA can serve as the phonon basis in the QPM.

The ground-state correlations are taken into account in describing the first quadrupole and octupole states in some spherical nuclei with incompletely filled shells. For example, the effect of ground-state correlations in $^{64-70}$Zn on the transition densities is significant for the first quadrupole and octupole states.[17]

Let us give the wave functions of one-phonon states and the wave functions containing one- and two-phonon terms, which will be needed to understand the results of the calculations without reference to Ref. 13. The Hamiltonian of the QPM consists of the axially symmetric Woods–Saxon potential describing the mean field, monopole pairing, and also isoscalar and isovector particle–hole (ph) and particle–particle (pp) multipole–multipole and spin–spin interactions, written in separable form.

The calculations are performed in the RPA using the wave function

$$Q^+_{\lambda\mu i\sigma}\Psi_0, \tag{1}$$

where

$$Q^+_{\lambda\mu i\sigma}=1/2\sum_{q_1q_2}\{\psi^{\lambda\mu i}_{q_1q_2}A^+(q_1q_2;\mu\sigma) - \phi^{\lambda\mu i}_{q_1q_2}A(q_1q_2;\mu(-\sigma))\} \tag{2}$$

is the operator for creation of a phonon of multipole order $\lambda\mu$, $i=1,2,3,\ldots$ is the number of the root of the secular equation of the RPA, and Ψ_0 is the ground-state wave function of the even–even nucleus. The quantum numbers of one-particle states are denoted by $q\sigma$, where $\sigma=\pm1$ and q is equal to K^π and the asymptotic quantum numbers $Nn_z\Lambda\uparrow$ for $K=\Lambda+1/2$ or $Nn_z\Lambda\downarrow$ for $K=\Lambda-1/2$. The QPM calculations are performed using a wave function containing one- and two-phonon terms:

$$\Psi_n(K_0^{\pi_0}\sigma_0)=\Bigg\{\sum_{i_0}R^n_{i_0}Q^+_{\lambda_0\mu_0 i_0\sigma_0} + 1/2\sum_{\substack{\lambda_1\mu_1 i_1\sigma_1\\ \lambda_2\mu_2 i_2\sigma_2}}\frac{(1+\delta_{\lambda_1\lambda_2}\delta_{\mu_1\mu_2}\delta_{i_1,i_2})^{1/2}}{[1+\delta_{K_0 0}(1-\delta_{\mu_1 0})]^{1/2}} \times\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0} \times P^n_{\lambda_1\mu_1 i_1\sigma_1,\lambda_2\mu_2 i_2\sigma_2}Q^+_{\lambda_1\mu_1 i_1\sigma_1}Q^+_{\lambda_2\mu_2 i_2\sigma_2}\Psi_0. \tag{3}$$

Here $n=1,2,3,\ldots$ labels states with a fixed value of $K_0^{\pi_0}$. Its normalization condition has the form

$$\sum_{i_0}(R^n_{i_0})^2+\sum_{\lambda_1\mu_1 i_1\geqslant\lambda_2\mu_2 i_2}(P^n_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2})^2 \times[1+\mathscr{K}^{K_0}(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)]=1. \tag{4}$$

The Pauli principle is taken into account in the two-phonon terms of the wave function (3) by introducing the function $\mathscr{K}^{K_0}(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)$, the explicit form of which is given in Refs. 13 and 15.

The improvement of the QPM consists of, first, the approximate elimination of the ghost state in the calculation of levels with $K^\pi=1^+$ and the use of spin–spin interactions together with quadrupole interactions in the RPA calculations, and, second, the RPA calculation of the energies and wave functions of states with $K^\pi=0^-$ and 1^- not only with the isoscalar and isovector ph and pp octupole interactions, but also with the isovector ph dipole–dipole interactions.

In earlier calculations[16,18,19] of excitations with $K^\pi=1^+$ the ghost state associated with nuclear rotation was not eliminated. In the new calculations whose results are presented here, the ghost state is approximately eliminated. This is done by choosing the constant κ_0^{21} of the isoscalar ph quadrupole interaction to be larger than its critical value $(\kappa_0^{21})_{\rm cr}$. At the value $(\kappa_0^{21})_{\rm cr}$ the first solution of the secular equation of the RPA vanishes. In Ref. 20 the procedure of eliminating the ghost state led to fixing of the constant $\kappa_0^{21}=(\kappa_0^{21})_{\rm cr}$. The following requirement is satisfied in our calculations:

$$\kappa_0^{21}>(\kappa_0^{21})_{\rm cr}. \tag{5}$$

The ghost state is defined as

$$|J_+\rangle=\frac{1}{N_{\rm sp}^{1/2}}|I_+^{\rm ph}\rangle \tag{6}$$

with normalization

$$\frac{1}{N_{\rm sp}}\langle I_-^{\rm ph}I_+^{\rm ph}\rangle=1, \tag{6'}$$

where

$$N_{\rm sp}=\sum_{\substack{q_1q_2\\K_1\geqslant K_2}}(u^{(-)}_{q_1q_2})^2[\langle q_2\sigma_0|I_-|q_1\sigma_0\rangle \times\langle q_1\sigma_0|I_+|q_2\sigma_0\rangle\delta_{K_1-K_2,1}+\langle q_2(-\sigma_0)|I_-|q_1\sigma_0\rangle \times\langle q_1\sigma_0|I_+|q_2(-\sigma_0)\rangle\delta_{K_1+K_2,1}],$$

$$N_{\rm sp}=N_{\rm sp}(\nu)+N_{\rm sp}(\pi). \tag{7}$$

The overlap of the wave function of the one-phonon state $Q^+_{21i\sigma_0}\Psi_0$ with the ghost state is

$$N^i_{spu}=\frac{1}{N_{\rm sp}}\langle J_-Q^+_{21i\sigma_0}\rangle\langle Q_{21i\sigma_0}J_+\rangle=\frac{1}{N_{\rm sp}}I_-^{21i}I_+^{21i}, \tag{8}$$

where

$$I_\pm^{21i}=\sum_\tau I_\pm^{21i}(\tau),$$

$$I_\pm^{21i}(\tau)=\sum_{\substack{q_1q_2\\K_1\geqslant K_2}}{}^\tau\langle q_1|j_\pm|q_2\rangle u^{(-)}_{q_1q_2}\psi^{21i}_{q_1q_2}. \tag{9}$$

We note that if the ghost state is not eliminated, we have the following normalization:

$$\sum_i N^i_{spu}=1. \tag{10}$$

When the condition (5) is satisfied, the ghost state is practically orthogonal to all the one-phonon states. For any one-phonon state the overlap with the ghost state is less than 0.005. The sum over the first 20 states $\Sigma_i^{20}N^i_{spu}$ has a value from 0.010 to 0.025. The sum over all the states in ^{164}Dy up to 30 MeV is 0.048. From this we see that the approximate elimination of the ghost state is quite satisfactory. The overlap of the ghost state with the excited states described by the wave functions $\Psi_n(K_0^{\pi_0}\sigma_0)$, consisting of one- and two-phonon parts, has the form

$$N^i_{spu}=\frac{1}{N_{\rm sp}}\langle J_-\Psi_n(K_0^{\pi_0}=1^+,\sigma_0)\rangle\langle\Psi_n^*(K_0^{\pi_0}=1^+,\sigma_0)J_+\rangle=\frac{1}{N_{\rm sp}}\sum_{ii'}R_i^nR_{i'}^nI_-^{21i}I_+^{21i'}. \tag{11}$$

The collective states with $K^\pi=1^+$ excited in (e,e') and (γ,γ') reactions were predicted by studying the two-rotor model.[8] It was assumed that the neutrons and protons exhaust the rotational oscillations about the axis perpendicular to the symmetry axis of the deformed nucleus. Comparison of the cross sections for the excitation of these states in (e,e') and (γ,γ') reactions has shown that they are excited via the orbital part of the M1-transition operator. The properties of these states are described microscopically in the RPA.[9–12]

The microscopic representation of the wave function of the scissors state is given in Ref. 11. It has the form

$$|\Psi_{\rm sc}\rangle=(N_{\rm sp}N_{\rm sp}(\nu)N_{\rm sp}(\pi))^{-1/2}[N_{\rm sp}(\pi)I_+^{\rm ph}(\nu)-N_{\rm sp}(\nu)I_+^{\rm ph}(\pi)], \tag{12}$$

where $N_{\rm sp}$ is given by Eq. (7), and $N_{\rm sp}(\nu)$ and $N_{\rm sp}(\pi)$ refer to the neutron and proton systems, respectively. The following normalization conditions hold:

$$\langle\Psi_{\rm sc}|\Psi_{\rm sc}\rangle=1, \tag{13}$$

$$\sum_i|\Psi_{\rm sc}|Q^+_{21i\sigma_0}\Psi_0\rangle|^2=1, \tag{14}$$

where the summation runs over all the solutions of the secular equation of the RPA.

The overlap of the wave function of the state (12) with the wave functions (1) and (3) has the form

$$\mathrm{Sc}^i=\langle Q_{21i}\Psi_{\rm sc}\rangle\langle\Psi^*_{\rm sc}Q^+_{21i}\rangle=\frac{[N_{\rm sp}(\pi)I_+^{21i}(\nu)-N_{\rm sp}(\nu)I_+^{21i}(\pi)]^2}{N_{\rm sp}N_{\rm sp}(\nu)N_{\rm sp}(\pi)}, \tag{15}$$

$$\mathrm{Sc}^n=\frac{1}{N_{\rm sp}N_{\rm sp}(\nu)N_{\rm sp}(\pi)}\sum_{ii'}R_i^nR_{i'}^n[N_{\rm sp}(\pi)I_+^{21i}(\nu)-N_{\rm sp}(\nu)I_+^{21i}(\pi)][N_{\rm sp}(\pi)I_+^{21i'}(\nu)-N_{\rm sp}(\nu)I_+^{21i'}(\pi)]. \tag{16}$$

3. REPRESENTATION OF THE RESULTS OF THE CALCULATIONS

In our calculations we used the single-particle energies and wave functions of the axially symmetric Woods–Saxon potential. The nuclear part of the Woods–Saxon potential consists of central and spin–orbit terms:

$$V_{\rm Nuc.}=V(r)+V_{ls}(r), \tag{17}$$

TABLE I. Parameters of the Woods-Saxon potential.

A	p/n	V_0 MeV	r_0 F	α F^{-1}	κ F^2
155	p	59.2	1.24	1.63	0.360
165	p	59.2	1.25	1.63	0.355
155	n	47.2	1.26	1.67	0.400
165	n	44.8	1.26	1.67	0.430

$$V(r)=\frac{-V_0^{N,Z}}{1+\exp\{\alpha[r-R(\theta,\phi)]\}}, \tag{18}$$

$$V_{ls}(r)=-\kappa(\vec{p}\times\vec{\sigma})\nabla V(r), \tag{19}$$

where $\vec{\sigma}$ is the Pauli matrix and $\vec{p}$ is the nucleon momentum. For the proton system it is necessary to add the Coulomb term

$$V_{\rm C}(r)=\frac{3}{4\pi}\frac{(Z-1)e^2}{R_0^3}\int\frac{n(r')dr'}{|r-r'|}, \tag{20}$$

where $n(r)$ is the charge density distribution in the nucleus:

$$n(r)=\{1+\exp[\alpha(r-R(\theta,\phi))]\}^{-1}.$$

The shape of the nucleus is described by the expression

$$R(\theta,\phi)=R_0\{1+\widetilde{\beta}+\beta_2Y_{20}(\theta,\phi)+\beta_4Y_{40}(\theta,\phi)\}, \tag{21}$$

where $R_0=r_0A^{1/3}$ is the radius of the equivalent spherical nucleus, and the constant $\widetilde{\beta}$ is often introduced to obtain the best conformity with the condition of conservation of the nuclear volume. Finally, β_2 and β_4 are the quadrupole $(\lambda=2)$ and hexadecapole $(\lambda=4)$ deformation parameters.

Since the single-particle energies and wave functions of the Woods–Saxon potential depend on the mass number A, the regions of deformed nuclei are split into zones so that the calculations do not have to be done for each value of A. The rare-earth region is split into the following A zones: 155, 165, 173, and 181. The fit of the parameters of the Woods–Saxon potential consists of the following four steps: (1) the set of potential parameters determined is used to calculate the single-particle energies and wave functions; (2) the equilibrium shape of the nucleus is calculated by the shell-correction method,[21] and the quadrupole and hexadecapole deformation parameters β_2 and β_4 are thereby fixed; (3) the phonons are calculated in the RPA; (4) the wave functions of odd nuclei are taken in the form of a sum of one-quasiparticle and quasiparticle⊗phonon components, the quasiparticle–phonon interactions are taken into account, and the energies and wave functions of the nonrotational states of odd nuclei are calculated and then compared with the corresponding experimental data. The agreement between the results of the calculations and experiment is improved by changing the parameters of the Woods–Saxon potential and then repeating the four steps of the calculations. This procedure is repeated until a sufficiently good description of the experimental data on the low-lying nonrotational levels of odd nuclei is obtained. The parameters of the Woods–Saxon potential are given in Table I.

The calculations were carried out using the single-particle energies and wave functions of the Woods–Saxon

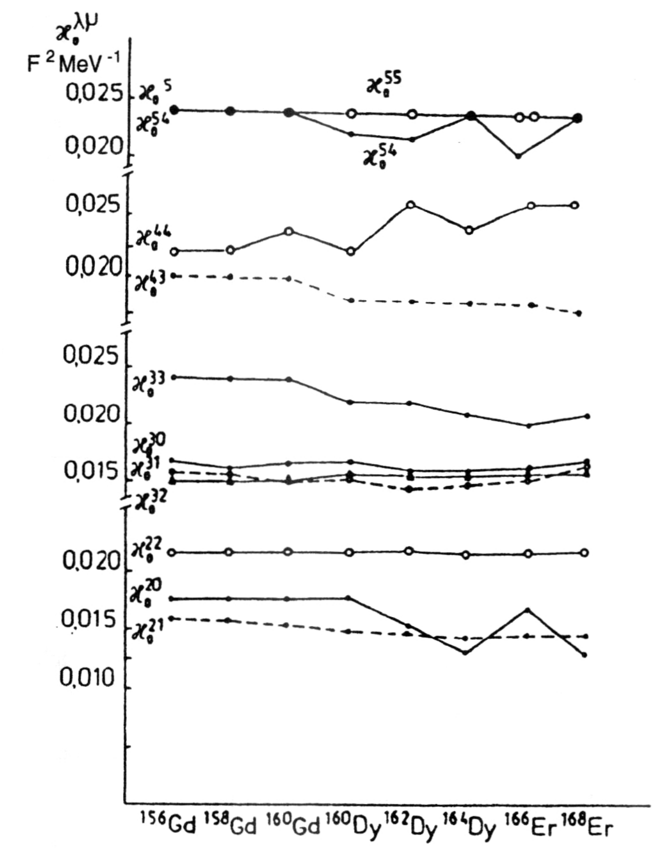

FIG. 1. Multipole-multipole isoscalar ph interaction constants $\kappa_0^{\lambda\mu}$ (in units of F^2 MeV^{-1}).

potential of the $A=155$ zone in ^{156,158,160}Gd with the equilibrium deformations $\beta_2=0.28$ and $\beta_4=0.04$, and of the $A=165$ zone in ^{160}Dy with $\beta_2=0.28$ and $\beta_4=0.02$, and in ^{162}Dy and ^{166,168}Er with $\beta_2=0.28$ and $\beta_4=-0.01$ and the nonaxiality parameter $\gamma=0$. All the single-particle levels from the bottom of the potential well up to $+5$ MeV were taken into account in calculating low-lying states with energy below 2.3 MeV.

The quasiparticle interaction constants were fixed as follows. The constant $\kappa_0^{\lambda\mu}$ of the isoscalar multipole ph interaction was taken to be that for which the calculated energy of the first $K^{\pi}_{n=1}$ state is close to the experimental value; the isovector constant was $\kappa_1^{\lambda\mu}=-1.5\kappa_0^{\lambda\mu}$, and the multipole pp interaction constant was $G^{\lambda\mu}=\kappa_0^{\lambda\mu}$. The isovector dipole ph interaction constant was $\kappa_1^{1\mu}=-1.5\kappa_0^{3\mu}$. The location of the giant isovector dipole resonance is described well for this value of the constant. The numerical values of the constants are given in Fig. 1. The constant κ_0^{21} is larger than that for which the first solution of the secular equation of the RPA is equal to zero. In this manner we approximately eliminated the ghost state with $K^{\pi}=1^+$ associated with rotation of the nucleus as a whole. The isoscalar and isovector spin–spin interaction constants are $\kappa_0^{011}=-0.0024$ F$^2\cdot$MeV^{-1} and $\kappa_1^{011}=-0.024$ F$^2\cdot$MeV^{-1}. The monopole pairing constant G_τ was fixed from the pairing energies, using the fact that $G^{20}=\kappa_0^{20}$. The energies of the two-quasiparticle poles were calculated taking into account monopole and quadrupole pairing, the blocking effect, and the Gallagher–Moszkowski corrections.[22]

Our phonon basis consists of ten ($i=1,2,3,\ldots,10$) phonons of each multipole order $\lambda\mu=20$, 22, 32, 33, 43, 44, 54, 55, and 65, and also twelve phonons of each order $\lambda\mu=21$, 30, and 31. In addition to the energies and wave functions, we calculated the reduced probabilities for Eλ and M1 transitions from the ground state $0^+0_{\text{g.s.}}$ to the excited state with $I=\lambda$ using the expressions given in Refs. 13 and 15. The reduced probabilities for Eλ and Mλ transitions between excited states were calculated using the expressions given in Refs. 13 and 23. We use a restricted space of one-particle states from the bottom of the potential well up to $+5$ MeV. Therefore, the calculations of the $B(\text{E}\lambda)$ for $\lambda=2$, 3, 4, and 5 were performed with the following effective proton and neutron charges: $e^{(\lambda)}_{\text{eff}}(p)=1.2$ and $e^{(\lambda)}_{\text{eff}}(n)=0.2$. The calculations with the complete basis and $e^{(\lambda)}_{\text{eff}}(p)=1$, $e^{(\lambda)}_{\text{eff}}(n)=0$ give roughly the same values of the $B(\text{E}\lambda)$. The reduced probabilities $B(\text{E}1)$ were calculated with the effective charges $e^{(1)}_{\text{eff}}(p)=N/A$ and $e^{(1)}_{\text{eff}}(n)=-Z/A$, and the $B(\text{M}\lambda)$ were calculated with $g_s^{\text{eff}}=0.7$.

We calculated the energies and wave functions of nonrotational states neglecting the Coriolis interaction. If necessary, it can be taken into account using the wave functions (3), as has been done in, for example, Ref. 16. The experimental data and results of the calculations are presented in the form of two tables for each nucleus. In the first we give the experimental and calculated energies and the values of $B(\text{E}\lambda)\uparrow$ for Eλ transitions with $\lambda>1$ from the ground state $0^+0_{\text{g.s.}}$ to excited states with fixed values of $I^{\pi}K_n$ with $\lambda=I$. The calculated structure of the nonrotational state is represented as the contribution (in percent) of the one-phonon $(\lambda\mu)_i$ and two-phonon $\{(\lambda_1\mu_1)_{i_1},(\lambda_2\mu_2)_{i_2}\}$ terms to the normalization of the wave function (2). Later in the table we give the contributions (in percent) of several of the largest two-quasineutron $\nu\nu$ and two-quasiproton $\pi\pi$ components to the normalization of the wave function of the one-phonon state $(\lambda\mu)_i$. In these tables we give all the nonrotational states with energy below 2.3 MeV. $B(\text{E}\lambda)\uparrow$ denotes the reduced probability for an Eλ transition from the ground state $0^+0_{\text{g.s.}}$ to an excited $I^{\pi}K_n$ state with $I=\lambda$, expressed in one-particle units:

$$B(E\lambda)\uparrow_{\text{one-p. units}}=\frac{2\lambda+1}{4\pi}\left(\frac{3}{3+\lambda}\right)^2(1.2A^{1/3})^{2\lambda}(\text{F})^{2\lambda}. \quad (22)$$

In the second table we give the E1 and M1 transitions from states with $K^{\pi}=0^-$, 1^-, and 1^+ to the ground state and E1, E2, and M1 transitions between excited states. The values of $B(\text{E}\lambda)$ and $B(\text{M}\lambda)$ are given in units of $e^2\text{F}^{2\lambda}$ and $\mu_N^2\text{F}^{2\lambda-2}$, respectively. The γ-transition probability is given in inverse seconds.

4. THE EXPERIMENTAL DATA AND RESULTS OF THE CALCULATIONS

The results of the calculations of the energies, wave functions, $B(\text{E}\lambda)\uparrow$, and the reduced probabilities for γ transitions between excited states are given in Tables II–XVII

TABLE II. Nonrotational states in ^{156}Gd.

	Experiment			QPM calculations				
K_n^π	E_n MeV	B(Eλ)↑ one-particle units	Structure	E_n MeV	B(Eλ)↑ one-particle units	Structure		%
0_1^+	1.049	0.63		1.2	0.8		$(20)_1$:88	
		1.38	$\tilde{S}(t,p)=0.01$		$\tilde{S}(t,p)=0.2$		$\{(22)_1,(22)_1\}$: 5	
							$\{(33)_1,(33)_1\}$: 1	
			$\tilde{S}(p,t)=0.10$		$\tilde{S}(p,t)=0.1$		$\{(20)_1,(20)_1\}$: 1	
			$\rho^2=0.051$		$\rho^2=0.004$	$(20)_1$:	$\nu\nu 521\uparrow-521\uparrow$	56
							$\pi\pi 411\uparrow-411\uparrow$	27
			(d,t)				$\nu\nu 651\uparrow-651\uparrow$	7
							$\pi\pi 413\downarrow$-$413\downarrow$	4
2_1^+	1.154	4.46		1.1	4.0		$(22)_1$: 96	
		2.8	(d,p)				$\{(20)_1,(22)_1\}$: 1	
							$\{(22)_2,(44)_1\}$: 1	
						$(22)_1$:	$\nu\nu 642\uparrow-660\uparrow$	20
							$\nu\nu 521\uparrow+521\downarrow$	13
							$\nu\nu 651\uparrow+660\uparrow$	12
							$\pi\pi 413\downarrow-411\downarrow$	12
0_2^+	1.168	0.31		1.8	0.1		$(20)_2$:93	
		0.32	$\tilde{S}(t,p)=0.23$		$\tilde{S}(t,p)=0.17$		$(20)_1$: 1; $(20)_5$: 2	
						$(20)_2$:	$\nu\nu 523\downarrow-523\downarrow$	25
					$\tilde{S}(p,t)=0.10$		$\nu\nu 521\uparrow-521\uparrow$	18
			$\rho^2=0.0037$		$\rho^2=0.002$		$\pi\pi 411\uparrow-411\uparrow$	17
			(d,t)(d,p)				$\nu\nu 651\uparrow-651\uparrow$	6
1_1^-	1.242	16.9		1.1	13		$(31)_1$:99	
		3.4				$(31)_1$:	$\pi\pi 532\uparrow-411\uparrow$	26
			(d,t)				$\nu\nu 642\uparrow-521\uparrow$	19
0_1^-	1.366	3.6		1.4	3.3		$(30)_1$:99	
						$(30)_1$:	$\nu\nu 521\uparrow-651\uparrow$	30
			(d,t)				$\pi\pi 532\uparrow-413\downarrow$	5
4_1^+	1.511	$\lvert g_k-g_R\rvert=0.5\pm0.1$		1.5	0.6		$(44)_1$:94	
		$\pi\pi 413\downarrow+411\uparrow$ large					$\{(22)_1,(22)_1\}$: 5	
						$(44)_1$:	$\pi\pi 413\downarrow+411\uparrow$	83
							$\nu\nu 642\uparrow+651\uparrow$	8
							$\nu\nu 523\downarrow+521\uparrow$	6
0_3^+	1.715		$\tilde{S}(t,p)=0.01$	1.8	0.1		$(20)_3$:90; $(20)_1$:3	
							$\{(22)_1,(22)_1\}$:3	
					$\tilde{S}(t,p)=0.02$	$(20)_3$:	$\pi\pi 413\downarrow-413\downarrow$	28
							$\pi\pi 411\uparrow-411\uparrow$	23
							$\nu\nu 651\uparrow-651\uparrow$	17
							$\nu\nu 523\downarrow-523\downarrow$	10
2_1^-	1.780			1.7	3.0		$(32)_1$: 98	
			(d,t)			$(32)_1$:	$\pi\pi 411\uparrow-523\uparrow$	51
							$\nu\nu 660\uparrow+521\uparrow$	23
							$\nu\nu 523\downarrow-660\uparrow$	5
2_2^+	1.828			1.9	0.1		$(22)_2$:86; $(22)_3$: 3	
							$\{(20)_1,(22)_1\}$: 3	
							$\{(22)_1,(44)_1\}$: 3	
							$(22)_2$: $\nu\nu 642\uparrow-660\uparrow$	74
							$\nu\nu 651\uparrow+660\uparrow$	15
							$\nu\nu 521\uparrow+521\downarrow$	5
0_4^+	1.851			2.2	0.01		$(20)_4$:91; $(20)_3$: 5	
							$\{(22)_1,(22)_1\}$: 1	
						$(20)_4$:	$\nu\nu 523\downarrow-523\downarrow$	64
							$\nu\nu 651\uparrow-651\uparrow$	12
							$\nu\nu 505\uparrow-505\uparrow$	8
4_2^+	1.861			1.9	0.02		$(44)_2$:90; $(44)_3$: 3	
			(d,t)(d,p)				$\{(22)_1,(22)_1\}$: 4	
						$(44)_2$:	$\nu\nu 523\downarrow+521\uparrow$	85
							$\pi\pi 413\downarrow+411\uparrow$	12
2_2^-	1.934			2.0	0.2		$(32)_2$:96	
			(d,t)(d,p)				$\{(22)_1,(54)_1\}$: 2	
						$(32)_2$:	$\nu\nu 521\uparrow+660\uparrow$	77
							$\pi\pi 523\uparrow-411\uparrow$	19
0_2^-	1.946			2.0	0.8		$(30)_2$: 96	
			(d,t)				$(30)_2$: $\pi\pi 532\uparrow-413\downarrow$	28
							$\nu\nu 521\uparrow-651\uparrow$	17
							$\nu\nu 523\downarrow-642\uparrow$	13

TABLE II. (*Continued.*)

	Experiment			QPM calculations				
K_n^π	E_n MeV	B(Eλ)↑ one-particle units	Structure	E_n MeV	B(Eλ)↑ one-particle units		Structure	%
1_1^+	1.966	0.16		1.9	0.04		$(21)_1$:99	
						$(21)_1$:	$\pi\pi 413\downarrow - 411\uparrow$	89
							$\nu\nu 642\uparrow - 651\uparrow$	8
1_2^+	2.027	0.43		2.0	0.9		$(21)_2$:93	
							$\{(20)_1,(21)_2\}$: 3	
						$(21)_2$:	$\nu\nu 642\uparrow - 651\uparrow$	63
							$\pi\pi 523\uparrow - 532\uparrow$	18
							$\pi\pi 413\downarrow - 411\uparrow$	10
							$\nu\nu 523\downarrow - 521\uparrow$	3
4_1^-	2.045			2.0	0.7		$(54)_1$:98	
						$(54)_1$:	$\nu\nu 651\uparrow + 523\downarrow$	60
							$\pi\pi 532\uparrow + 411\uparrow$	20
1_2^-				2.0	0.4		$(31)_2$:96	
							$\{(20)_1,(31)_2\}$	2
						$(31)_2$:	$\nu\nu 642\uparrow - 521\uparrow$	76
							$\pi\pi 532\uparrow - 411\uparrow$	19
7_1^-	2.138			2.8			$\nu\nu 505\uparrow + 651\uparrow$	100
1_3^+	2.187			2.3	0.07		$(21)_3$:99	
						$(21)_3$:	$\nu\nu 651\uparrow - 660\uparrow$	85
							$\nu\nu 642\uparrow - 651\uparrow$	10
3_1^-				2.1	0.2		$(33)_1$:89;$(33)_2$: 6	
						$(33)_1$:	$\nu\nu 521\uparrow + 651\uparrow$	93
							$\pi\pi 514\uparrow - 411\uparrow$	2
4_3^+				2.1	0.01		$(44)_3$:84;$(44)_2$: 3	
							$\{(22)_1,(22)_1\}$: 8	
						$(44)_3$:	$\pi\pi 413\downarrow + 411\uparrow$	94
							$\nu\nu 523\downarrow + 521\uparrow$	4
0_3^-				2.4	0.8		$(30)_3$:93; $(30)_2$: 1	
							$\{(20)_1,(30)_3\}$: 2	
						$(30)_3$:	$\pi\pi 532\uparrow - 413\downarrow$	31
							$\nu\nu 523\downarrow - 642\uparrow$	18
3_1^+				2.3	4.0		$(43)_1$:97	
						$(43)_1$:	$\nu\nu 642\uparrow + 660\uparrow$	16
							$\nu\nu 532\downarrow + 521\uparrow$	13
2_3^+				2.3	0.3		$(22)_3$:80;$(22)_2$: 5	
							$\{(22)_1,(44)_1\}$: 5	
							$(22)_3$: $\nu\nu 651\uparrow + 660\uparrow$	61
							$\nu\nu 521\uparrow + 521\downarrow$	36
0_5^+				2.3	0.01		$(20)_5$:95;$(20)_4$: 2	
							$\{(31)_1,(31)_1\}$: 1	
						$(20)_5$:	$\nu\nu 642\uparrow - 642\uparrow$	41
							$\pi\pi 413\downarrow - 413\downarrow$	7

along with the corresponding experimental data. The results of the calculations in Refs. 16, 18, 19, and 23–26 are quoted in these tables together with the results of the new calculations. We give the ratios of the experimental and calculated spectroscopic factors of (t,p) and (p,t) reactions for transitions to excited 0_n^+ states and transitions between ground states, i.e., $\widetilde{S}_n(t,p) = S_n(t,p)/S_{\text{g.s.}}(t,p)$, $\widetilde{S}_n(p,t) = S_n(p,t)/S_{\text{g.s.}}(p,t)$. The experimental values of the two-quasineutron $\nu\nu$ or two-quasiproton $\pi\pi$ components of the wave functions of the levels excited in one-nucleon transfer reactions and in β decay are given. The notations (d,t), (d,p), (^{3}He, α), and so on indicate high intensity of the corresponding reactions. The experimental and calculated values of ρ^2 were found from the matrix elements of E0 transitions, and the values of X(E0/E2) were obtained from the reduced probabilities of E0 and E2 transitions.

The experimental data for ^{156,158,160}Gd in Tables II–VII are taken from Refs. 27–39, and the lower values for

TABLE III. E1 and M1 transitions to the ground state and E1, E2, and M1 transitions between excited states in ^{156}Gd.

Initial state $I^{\pi}K_n$	Initial state E_n, MeV	Eλ or Mλ	Final state n_f	Final state $I^{\pi}K_n$	Final state E_n, MeV	$B(E\lambda)\downarrow$, $e^2F^{2\lambda}$ or $B(M\lambda)\downarrow$, $\mu_N^2F^{2\lambda-2}$ exp. [Ref.]	$B(E\lambda)\downarrow$, $e^2F^{2\lambda}$ or $B(M\lambda)\downarrow$, $\mu_N^2F^{2\lambda-2}$ calculation
1^-1_1	1.242	E1	1	$0^+0_{g.s.}$	0	$3\cdot10^{-3}$ 35	$30\cdot10^{-3}$
						$1\cdot10^{-3}$ 29	
		E1	2	0^+0_1	1.049	$2\cdot10^{-4}$ 29	$5\cdot10^{-6}$
1^-0_1	1.366	E1	1	$0^+0_{g.s.}$	0	$5\cdot10^{-3}$ 35	$40\cdot10^{-3}$
						$2.6\cdot10^{-3}$ 29	
		E1	2	2^+0_1	1.129	$7\cdot10^{-4}$ 29	$3\cdot10^{-5}$
4^+0_1	1.298	E2	1	$2^+0_{g.s.}$	0.089	61 29	55
						46 30	
		E2	2	2^+2_1	1.154		3
4^+4_1	1.511	E2	1	2^+2_1	1.154		64
0^+0_3	1.715	E2	1	2^+2_1	1.154		28
		E1	2	1^-1_1	1.242		$3\cdot10^{-4}$
		E1	3	1^-0_1	1.366		$2\cdot10^{-5}$
2^+0_3	1.771	E2	1	$2^+0_{g.s.}$	0.089	90 29	8
		E1	2	1^-1_1	1.242	$5\cdot10^{-5}$ 29	$4\cdot10^{-5}$
		E1	3	1^-0_1	1.366	$1.6\cdot10^{-4}$ 29	$7\cdot10^{-6}$
2^-2_1	1.780	E1	1	2^+2_1	1.154	$1.8\cdot10^{-3}$ 29	$2\cdot10^{-3}$
		M1	2	2^-2_1	1.320	$8\cdot10^{-3}$ 29	0.02
2^+2_2	1.828	E2	1	$2^+0_{g.s.}$	0.089		6
		E2	2	2^+0_1	1.129		15
		M1	3	2^+2_1	1.154		0.003
		E2	4	2^+0_2	1.258		2
0^+0_4	1.851	E2	1	2^+0_1	1.129		3
		E2	2	2^+2_1	1.154		6
		E1	3	1^-1_1	1.242		$4\cdot10^{-4}$
0^-0_2	1.946	E1		$0^+0_{g.s.}$	0		$8\cdot10^{-3}$
1^+1_1	1.966	M1		$0^+0_{g.s.}$	0		$1\cdot10^{-3}$
1^+1_2	2.027	M1		$0^+0_{g.s.}$	0	0.06 35	0.26
4^+4_2	1.861	E2	1	2^+2_1	1.154		24
		M1	2	4^+4_1	1.511		0.04
		E2		4^+4_1	1.511		0.2
2^-2_2	1.934	M1	1	2^-1_1	1.320		$3\cdot10^{-3}$
		E2	2	3^-0_1	1.468		0.4
$3^-(1_2)$	1.9344	E1	1	$2^+0_{g.s.}$	0.089	$5\cdot10^{-5}$ 29	$20\cdot10^{-5}$
		E1	2	2^+0_1	1.129	$6\cdot10^{-5}$ 29	$4\cdot10^{-6}$
		E2	3	1^-1_1	1.242	290 29	0.3
		E1	4	3^+2_1	1.248	$1.2\cdot10^{-4}$ 29	$2\cdot10^{-7}$
		E1	5	2^+0_2	1.258	$(5\cdot10^{-5})$ 29	$4\cdot10^{-6}$
		M1	6	2^-2_1	1.780	$(27\cdot10^{-3})$ 29	0.002
1^-0_2	1.946	E1	1	$0^+0_{g.s.}$	0	$(8\pm3)10^{-4}$ 35	$60\cdot10^{-4}$
						$4.7\cdot10^{-4}$ 29	
		M1	2	1^-1_1	1.242		0.01
		E1	3	2^+0_2	1.258	$9\cdot10^{-4}$ 29	$3\cdot10^{-5}$
1^+1_1	1.966	E2	1	$2^+0_{g.s.}$	0.089	20 33	13
		M1	2	2^+2_1	1.154		0.02
		E1	3	1^-1_1	1.242		$8\cdot10^{-5}$
		E1	4	1^-0_1	1.366		$2\cdot10^{-6}$
1^+1_2	2.027	E2	1	$2^+0_{g.s.}$	0.089	55 33	100
						120 29	
		M1	2	2^+0_1	1.129	0.04 29	10^{-4}
		M1	3	0^+0_2	1.168	0.05 29	0.01
		E1	4	2^-1_1	1.320	0.038 29	$2\cdot10^{-4}$
4^-4_1	2.045	E1	1	4^+4_1	1.511		$3\cdot10^{-5}$

$B(E\lambda)\uparrow$ in Tables II, IV, and VI are taken from Ref. 39. The experimental data for ^{160}Dy in Tables VIII and IX are taken from Refs. 30, 37, and 40–49. In Tables X and XI we give the experimental data for ^{162}Dy obtained in Ref. 19 and also taken from Refs. 50–52. The experimental data for ^{164}Dy in Tables XII and XIII were obtained in Refs. 45, 47, 49, 51, and 53–58. The experimental data for ^{166}Er in Tables XIV andXV are taken from Refs. 49 and 59–64. The experimental data for ^{168}Er in Tables XVI and XVII are taken from Refs. 49 and 65–75.

TABLE IV. Nonrotational states of ^{158}Gd.

	Experiment				QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	%	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
1_1^-	0.977	11.9			1.0	8.0		$(31)_1$:98	
		8						$\{(20)_1,(31)_1\}$: 1	
			(t,α): $\pi\pi 532\uparrow-411\uparrow$	45			$(31)_1$:	$\nu\nu 642\uparrow-521\uparrow$	46
			(d,p): $\nu\nu 642\uparrow-521\uparrow$	40				$\pi\pi 532\uparrow-411\uparrow$	25
2_1^+	1.187	3.4			1.2	3.6		$(22)_1$: 95	
								$\{(22)_1,(44)_1\}$:1	
							$(22)_1$:	$\nu\nu 521\uparrow+521\downarrow$	21
								$\nu\nu 523\downarrow-521\downarrow$	13
								$\pi\pi 411\uparrow+411\downarrow$	12
								$\pi\pi 413\downarrow-411\downarrow$	10
								$\nu\nu 642\uparrow-660\uparrow$	8
0_1^+	1.196	0.31			1.0	0.4		$(20)_1$:94	
			(d,p): $\nu\nu 521\uparrow-521\uparrow$					$\{(20)_1,(20)_1\}$: 1.4	
			large			$\rho^2=3\cdot10^{-3}$	$(20)_1$:	$\nu\nu 521\uparrow-521\uparrow$	24
		$\rho^2=(7.2\pm2.1)\cdot10^{-4}$				$X(E0/E2)=0.03$		$\nu\nu 523\downarrow-523\downarrow$	18
						$\widetilde{S}(p,t)=0.26$		$\pi\pi 411\uparrow-411\uparrow$	16
								$\nu\nu 505\uparrow-505\uparrow$	15
								$\nu\nu 402\downarrow-402\downarrow$	10
0_1^-	1.263	2.2			1.3	3.0		$(30)_1$: 97	
		2.7						$\{(20)_1,(30)_1\}$: 1	
							$(30)_1$:	$\nu\nu 642\uparrow-523\downarrow$	30
4_1^+	1.380		(t,α): $\pi\pi 413\downarrow+411\uparrow$		1.4	0.6		$(44)_1$: 96	
			large					$\{(22)_1,(22)_1\}$: 2	
							$(44)_1$:	$\pi\pi 413\downarrow+411\uparrow$	80
								$\nu\nu 523\downarrow+521\uparrow$	13
								$\nu\nu 642\uparrow+651\uparrow$	4
0_2^+	1.452	0.37	$\rho^2=0.032$		1.6	0.2		$(20)_2$: 93; $(20)_1$: 1	
		$X(E0/E2)=0.97\pm0.12$						$\{(33)_1,(33)_1\}$: 2	
						$\rho^2=0.03$	$(20)_2$:	$\pi\pi 413\downarrow-413\downarrow$	40
			$\widetilde{S}(p,t)=0.23$			$X(E0/E2)=0.97$		$\nu\nu 521\uparrow-521\uparrow$	20
								$\nu\nu 505\uparrow-505\uparrow$	15
						$\widetilde{S}(p,t)=10^{-4}$		$\pi\pi 411\uparrow-411\uparrow$	10
4_1^-	1.636		(d,p): $\nu\nu 521\uparrow+642\uparrow$	72	1.7	0.3		$(54)_1$: 98	
			(t,α): $\pi\pi 532\uparrow+411\uparrow$				$(54)_1$:	$\nu\nu 521\uparrow+642\uparrow$	73
			significant					$\pi\pi 532\uparrow+411\uparrow$	11
0_3^+	1.743		(t,α): $\pi\pi 411\uparrow-411\uparrow$		1.8	0.02		$(20)_3$: 97	
			significant				$(20)_3$:	$\pi\pi 411\uparrow-411\uparrow$	40
								$\nu\nu 521\uparrow-521\uparrow$	40
								$\nu\nu 523\downarrow-523\downarrow$	15
2_1^-	1.794	5.2			1.8	3.5		$(32)_1$: 98	
							$(32)_1$:	$\pi\pi 523\uparrow-411\uparrow$	58
								$\nu\nu 633\uparrow-521\uparrow$	12
1_1^+	1.848		(t,α): $\pi\pi 413\downarrow-411\uparrow$		1.8	0.01		$(21)_1$: 99	
			large				$(21)_1$:	$\pi\pi 413\downarrow-411\uparrow$	90
1_2^-	1.856				1.8	1.2		$(31)_2$: 98	
							$(31)_2$:	$\nu\nu 642\uparrow-521\uparrow$	48
								$\pi\pi 532\uparrow-411\uparrow$	42
5_1^-					1.9	0.1		$(55)_1$: 97	
								$\{(20)_1,(55)_1\}$:2	
							$(55)_1$:	$\nu\nu 523\downarrow+642\uparrow$	98
4_2^+	1.920		(d,p): $\nu\nu 523\downarrow+521\uparrow$	~75	1.9	0.002		$(44)_2$: 95	
								$\{(22)_1,(22)_1\}$: 2	
							$(44)_2$:	$\nu\nu 523\downarrow+521\uparrow$	83
								$\pi\pi 413\downarrow+411\uparrow$	16
1_2^+	1.930		(t,α): $\pi\pi 413\downarrow-411\uparrow$		1.9	0.003		$(21)_2$: 98	
			significant				$(21)_2$:	$\nu\nu 523\downarrow-521\uparrow$	70
								$\nu\nu 642\uparrow-651\uparrow$	9
								$\pi\pi 523\uparrow-532\uparrow$	8
								$\pi\pi 413\downarrow-411\uparrow$	6
0_2^-					2.0	1.5		$(30)_2$: 97	
							$(30)_2$:	$\nu\nu 523\downarrow-642\uparrow$	18
								$\nu\nu 521\uparrow-651\uparrow$	14
								$\pi\pi 532\uparrow-413\downarrow$	3

TABLE IV. (*Continued.*)

	Experiment				QPM calculations				
K^π_n	E_n MeV	B(Eλ)↑ one-particle units	Structure	%	E_n MeV	B(Eλ)↑ one-particle units		Structure	%
0^+_4	(1.952)				2.0	0.3		$(20)_4$: 94; $(20)_2$: 1	
								$\{(22)_1,(22)_1\}$: 3	
							$(20)_4$:	$\pi\pi 413\downarrow - 413\downarrow$	55
								$\nu\nu 523\downarrow - 523\downarrow$	12
								$\nu\nu 505\uparrow - 505\uparrow$	5
								$\pi\pi 532\uparrow - 532\uparrow$	5
1^+_3					2.3	0.2		$(21)_3$: 94	
								$\{(21)_3,(20)_1\}$ 3	
							$(21)_3$:	$\nu\nu 523\downarrow - 521\uparrow$	30
								$\nu\nu 642\uparrow - 651\uparrow$	22
								$\pi\pi 523\uparrow - 532\uparrow$	17
3^-_1					2.1	3.8		$(33)_1$: 90	
								$\{(20)_1,(33)_1\}$: 2	
							$(33)_1$:	$\pi\pi 514\uparrow - 411\uparrow$	21
								$\nu\nu 521\uparrow + 651\uparrow$	12
								$\pi\pi 523\uparrow - 420\uparrow$	12
4^-_2	2.176		(t,α): $\pi\pi 532\uparrow + 411\uparrow$		2.1	0.2		$(54)_1$: 98	
	$I=5$			large			$(54)_1$:	$\pi\pi 532\uparrow + 411\uparrow$	54
								$\nu\nu 521\uparrow + 642\uparrow$	24
								$\nu\nu 523\downarrow + 651\uparrow$	18
3^+_1					2.2	4.0		$(43)_1$: 97	
								$\{(30)_1,(33)_1\}$: 1	
							$(43)_1$:	$\nu\nu 642\uparrow + 660\uparrow$	13
6^-_1					2.2			$\pi\pi 413\downarrow + 523\uparrow$	100
4^+_3					2.2	0.01		$(44)_3$: 89	
								$\{(22)_1,(22)_1\}$: 4	
								$\{(20)_1,(44)_3\}$: 3	
							$(44)_3$:	$\nu\nu 642\uparrow + 651\uparrow$	90
2^+_2					2.3	0.1		$(22)_2$: 83; $(22)_3$: 2	
								$\{(22)_1,(20)_1\}$: 2	
								$\{(20)_1,(22)_2\}$: 2	
								$\{(22)_1,(44)_2\}$: 3	
							$(22)_2$:	$\nu\nu 642\uparrow - 660\uparrow$	40
								$\nu\nu 521\uparrow + 521\downarrow$	28
								$\nu\nu 523\downarrow - 521\downarrow$	18
								$\pi\pi 411\uparrow + 411\downarrow$	4

5. NONROTATIONAL STATES AND γ-TRANSITION PROBABILITIES

5.1. General remarks

The energies and wave functions of quadrupole states with $\lambda\mu=20$ and 22 and octupole states with $\lambda\mu=30$, 31, and 32 in the region $150\leqslant A\leqslant 184$ calculated in the RPA in 1965 using the single-particle energies and wave functions of the Nilsson potential are given in Refs. 76 and 77. The amplitudes $\psi^{\lambda\mu 1}_{q_1q_2}$ and $\phi^{\lambda\mu 1}_{q_1q_2}$ of the wave functions of the first quadrupole and octupole one-phonon states are given in Ref. 76. A series of wave functions with large amplitude were found in one-nucleon transfer reactions. The values of B(E2) and B(E3) for the excitation of the first quadrupole and octupole states are given in Ref. 77. For many years these calculations served as the reference for experimental studies.

The energies and wave functions of two-quasiparticle states and the first two-phonon states with $K^\pi=0^+$, 2^+, 0^-, 1^-, and 2^- calculated in the RPA with ph isoscalar interactions using the single-particle energies and wave functions of the Woods–Saxon potential are given in Ref. 4. The six largest two-quasiparticle components of the wave functions of one-phonon states are given for each one. Many of the predictions made in Ref. 4 were later confirmed experimentally.

The energies and wave functions of hexadecapole states with $K^\pi=3^+$ and 4^+ in even–even deformed nuclei in the region $158\leqslant A\leqslant 188$ were calculated in the RPA in Ref. 78. It was shown that there are collective hexadecapole and two-quasiparticle states among the low-lying states with $K^\pi=3^+$ and 4^+. The features of quadrupole and hexadecapole states are described in Ref. 79. The effect of interactions of high multipole order with $\lambda=5$, 6, 7, and 9 on the mixing of two-quasineutron and two-quasiproton states with large values of K in even–even deformed nuclei was studied in Ref. 80. The description of the experimental data on the mixing of two-quasineutron and two-quasiproton configurations in ^{176,178}Hf, ^{171}Yb, ^{168}Er, and ^{158}Gd obtained in the RPA is qualitatively correct. These studies show that in describing the structure of deformed nuclei it is also necessary

TABLE V. E1 and M1 transitions to the ground state and E1, E2, and M1 transitions between excited states in ^{158}Gd.

Initial state		Eλ or Mλ	Final state			$B(E\lambda)\downarrow$, $e^2F^{2\lambda}$ or $B(M\lambda)\downarrow$, $\mu_N^2F^{2\lambda-2}$	
$I^\pi K_n$	E_n, MeV		n_f	$I^\pi K_n$	E_n, MeV	exp. [Ref.]	calculation
1^-1_1	0.977	E1	1	$0^+0_{\text{g.s.}}$	0		$15\cdot10^{-3}$
0^+0_1	1.196	E2	1	$2^+0_{\text{g.s.}}$	0.079	80.1±5.6 30	100
		E1	2	1^-1_1	0.977	$1.23\cdot10^{-4}$ 30	$2\cdot10^{-4}$
4^+0_1	1.407	E2	1	$2^+0_{\text{g.s.}}$	0.079	22.9 30	30
		E1	2	3^-1_1	1.041		$4\cdot10^{-5}$
		E2	3	2^+2_1	1.187		0.34
1^-0_1	1.263	E1	1	$0^+0_{\text{g.s.}}$	0	$6.6\cdot10^{-3}$ 35	$20\cdot10^{-3}$
4^+4_1	1.381	E2	1	2^+2_1	1.187		50
2^+0_2	1.517	E2	1	$0^+0_{\text{g.s.}}$	0	18.7 30	10
		E1	2	1^-1_1	0.977	$5\cdot10^{-4}$ 30	$2\cdot10^{-5}$
		E2	3	2^+2_1	1.187		6
		E1	4	1^-0_1	1.263	$3\cdot10^{-4}$ 30	$5\cdot10^{-5}$
4^-4_1	1.636	M2	1	3^+2_1	1.265		0.03
		E1	2	4^+4_1	1.380		10^{-5}
2^+0_3	1.792	E2	1	$4^+0_{\text{g.s.}}$	0.261		6
		E1	2	1^-1_1	0.977		$2\cdot10^{-6}$
		E2	3	2^+0_1	1.259		0.06
		E1	4	1^-0_1	1.263		$5\cdot10^{-5}$
2^-2_1	1.794	M1	1	2^-1_1	1.023		0.004
		E1	2	2^+2_1	1.187		$3\cdot10^{-3}$
1^+1_1	1.848	M1	1	$0^+0_{\text{g.s.}}$	0		$3\cdot10^{-5}$
		E1	2	2^-1_1	1.023		$8\cdot10^{-5}$
		E1	3	1^-0_1	1.263		$2\cdot10^{-6}$
1^-1_2	1.856	E1	1	$0^+0_{\text{g.s.}}$	0		$4\cdot10^{-4}$
		M1	2	1^-1_1	0.977		0.06
		E1	3	2^+2_1	1.187		$7\cdot10^{-5}$
		M1	4	1^-0_1	1.263		0.03
4^+4_2	1.920	E2	1	2^+2_1	1.187		20
		M1	2	4^+4_1	1.380		0.04
		E1	3	4^-4_1	1.636		$2\cdot10^{-4}$
1^+1_2	1.930	M1	1	$0^+0_{\text{g.s.}}$	0		$1\cdot10^{-3}$
		E1	2	1^-1_1	0.977		$4\cdot10^{-4}$
		M1	3	2^+2_1	1.187		0.07

to take into account multipole interactions with $\lambda>3$.

The calculations of the nonrotational states in even–even deformed nuclei given in the present review differ significantly from the calculations performed earlier[3–6,76–78] in the following respects.

(1) More complicated secular equations of the RPA are used. In the earlier calculations only isoscalar multipole–multipole ph interactions were taken into account, whereas here isoscalar and isovector multipole–multipole ph and pp interactions are included. Spin–spin interactions are also taken into account in describing one-phonon states with $K^\pi=1^+$. Isovector dipole–dipole ph interactions are included in describing states with $K^\pi=0^-$ and 1^-.

(2) The wave functions of the nonrotational states, which consist of one- and two-phonon terms, have the form (3).

(3) All the constants are fixed in the construction of the phonon basis. There are no free parameters in the calculations using the wave function (3).

Another difference is that we have calculated the reduced probabilities for Eλ and Mλ transitions between excited states, in addition to those for Eλ transitions from the ground to excited states. Calculations were performed for all the nonrotational states with excitation energy below 2.3 MeV.

5.2. 0^+ states

Let us consider excited 0^+ states. For many years the first 0^+_{I} state was treated as a beta-vibrational state[2] and described in the RPA with quadrupole ph interaction and interaction of the superconductor type.[3,4,76,77] The energies of the first excited 0^+_{I} states were described correctly in Refs. 4 and 76, and the $B(\mathrm{E2})\uparrow$ were greatly overestimated, as was found after measuring them. The experiments performed during the last 20 years have shown that in deformed nuclei of the rare-earth region the first 0^+_{I} states cannot be treated as beta-vibrational states, owing to the small values of the reduced probabilities for E2 transitions to the rotational band constructed on the ground state. In addition, the density of low-lying 0^+_{I} states discovered experimentally has turned out to be larger than the calculated value.

Significant progress in the description of 0^+ excited states in even–even deformed nuclei was made in Ref. 81, where quadrupole pp interactions were included along with

TABLE VI. Nonrotational states in ^{160}Gd.

	Experiment			QPM calculations				
K^{π}_{n}	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
2^+_1	0.988	2.8		0.9	2.7		$(22)_1$:97	
							$\{(22)_1,(44)_1\}$: 1	
						$(22)_1$:	$\nu\nu 521\uparrow+521\downarrow$	25
							$\nu\nu 523\downarrow-521\downarrow$	42
							$\pi\pi 411\uparrow+411\downarrow$	6
4^+_1	1.070			1.17	0.6		$(44)_1$:98	
							$\{(22)_1,(22)_1\}$: 1	
						$(44)_1$:	$\nu\nu 523\downarrow+521\uparrow$	51
							$\pi\pi 413\downarrow+411\uparrow$	45
0^-_1	1.224	11.8 3.1		1.3	5.0		$(30)_1$:99	
						$(30)_1$:	$\nu\nu 523\downarrow-642\uparrow$	30
							$\pi\pi 532\uparrow-413\downarrow$	4
(0^+)	(1.326)							
0^+_1	1.380		$\tilde{S}(t,p)=0.14$	1.3	0.3		$(20)_1$:84; $(20)_2$:3	
							$(20)_3$:6	
							$\{(33)_1,(33)_1\}$: 2	
							$\{(33)_1,(33)_2\}$: 2	
						$(20)_1$:	$\nu\nu 523\downarrow-523\downarrow$	75
					$\tilde{S}(t,p)=0.24$		$\pi\pi 411\uparrow-411\uparrow$	9
							$\nu\nu 521\downarrow-521\downarrow$	6
1^-_1				1.5	2.1		$(31)_1$:99	
						$(31)_1$:	$\nu\nu 642\uparrow-521\uparrow$	71
							$\pi\pi 532\uparrow-411\uparrow$	12
3^-_1	(1.462)			1.5	2.7		$(33)_1$:86	
							$\{(20)_3,(33)_1\}$: 2	
						$(33)_1$:	$\nu\nu 642\uparrow+521\downarrow$	54
							$\pi\pi 514\uparrow-411\uparrow$	10
4^+_2	(1.531)			1.5	0.1		$(44)_2$:99	
						$(44)_2$:	$\pi\pi 413\downarrow+411\uparrow$	52
							$\nu\nu 523\downarrow+521\uparrow$	47
2^+_2	(1.584)			1.8	0.2		$(22)_2$:95	
							$\{(22)_1,(44)_1\}$: 1	
						$(22)_2$:	$\nu\nu 523\downarrow-521\downarrow$	50
							$\nu\nu 521\uparrow+521\downarrow$	45
							$\pi\pi 411\uparrow+411\downarrow$	2
0^+_2				1.6	0.6		$(20)_2$:64;$(20)_3$:20	
							$(20)_1$:11	
							$\{(33)_1,(33)_1\}$: 2	
							$\{(33)_1,(33)_2\}$: 2	
					$\tilde{S}(t,p)=0.04$	$(20)_2$:	$\nu\nu 521\uparrow-521\uparrow$	61
							$\nu\nu 523\downarrow-523\downarrow$	23
							$\pi\pi 411\uparrow-411\uparrow$	8
2^-_1				1.6	2.8		$(32)_1$:98	
						$(32)_1$:	$\nu\nu 633\uparrow-521\uparrow$	41
							$\pi\pi 523\uparrow-411\uparrow$	33
							$\nu\nu 642\uparrow-521\downarrow$	9
5^-_1				1.6	0.01		$(55)_1$:89	
						$(55)_1$:	$\nu\nu 642\uparrow+523\downarrow$	99
4^-_1				1.7	0.2		$(54)_1$:99	
						$(54)_1$:	$\nu\nu 642\uparrow+521\uparrow$	93
							$\pi\pi 411\uparrow+532\uparrow$	2
3^-_2	(1.688)			1.9	2.7		$(33)_2$:85	
							$\{(20)_1,(30)_1\}$:2	
						$(33)_2$:	$\nu\nu 642\uparrow+521\downarrow$	46
							$\pi\pi 514\uparrow-411\uparrow$	13
0^+_3				1.8	0.01		$(20)_3$:65;$(20)_2$:30	
							$\{(33)_1,(33)_2\}$: 1	
					$\tilde{S}(t,p)=10^{-3}$	$(20)_3$:	$\pi\pi 411\uparrow-411\uparrow$	46
							$\pi\pi 413\downarrow-413\downarrow$	33
1^+_1				1.9	10^{-4}		$(21)_1$:99	
						$(21)_1$:	$\nu\nu 523\downarrow-521\uparrow$	99
1^+_2				2.0	10^{-3}		$(21)_2$:99	
						$(21)_2$:	$\pi\pi 411\uparrow-411\downarrow$	99
0^-_2	1.967			1.9	1.7		$(30)_2$:99	

TABLE VI. (*Continued.*)

	Experiment			QPM calculations				
K_n^π	E_n MeV	B(Eλ)↑ one-particle units	Structure	E_n MeV	B(Eλ)↑ one-particle units		Structure	%
						$(30)_2$:	$\nu\nu 523\downarrow - 642\uparrow$	18
							$\pi\pi 413\downarrow - 532\uparrow$	17
1_2^-	(1.997)	$\log ft = 5.2$ from ^{160}Eu 0^-		2.0	1.2		$(31)_2$:99	
						$(31)_2$:	$\pi\pi 523\uparrow - 413\downarrow$	39
		$\pi\pi 523\uparrow - 413\downarrow$	large				$\pi\pi 532\uparrow - 411\uparrow$	28
							$\nu\nu 642\uparrow - 521\uparrow$	23
4_2^-				2.0	0.3		$(54)_2$:99	
						$(54)_2$:	$\pi\pi 523\uparrow + 411\downarrow$	83
							$\nu\nu 521\uparrow + 642\uparrow$	5
2_2^-				2.1	0.02		$(32)_2$:98	
						$(32)_2$:	$\nu\nu 642\uparrow - 521\downarrow$	88
							$\nu\nu 633\uparrow - 521\uparrow$	10
0_4^+	2.236			2.0	10^{-4}		$(20)_4$:96; $(20)_3$: 1	
			$\widetilde{S}(t,p)=0.18$		$\widetilde{S}(t,p)=0.19$	$(20)_4$:	$\nu\nu 521\downarrow - 521\downarrow$	29
							$\nu\nu 642\uparrow - 642\uparrow$	14
							$\nu\nu 633\uparrow - 633\uparrow$	11
2_3^+	(1.996)			2.1	0.2		$(22)_3$:14; $(22)_4$: 2	
							$\{(22)_1,(44)_1\}$:78	
						$(22)_3$:	$\pi\pi 411\uparrow + 411\downarrow$	27
							$\nu\nu 521\uparrow + 521\downarrow$	26
							$\pi\pi 413\downarrow - 411\downarrow$	19
1_3^+				2.2	0.003		$(21)_3$:98	
						$(21)_3$:	$\nu\nu 521\uparrow - 521\downarrow$	97
1_4^+	(2.348)			2.4	0.2		$(21)_4$:98	
						$(21)_4$:	$\nu\nu 633\uparrow - 642\uparrow$	73
							$\pi\pi 523\uparrow - 532\uparrow$	14
1_3^-				2.2	0.2		$(31)_3$:97	
							$\{(21)_1,(32)_1\}$: 2	
						$(31)_3$:	$\pi\pi 523\uparrow - 413\downarrow$	91
							$\pi\pi 532\uparrow - 411\uparrow$	8
5_2^-				2.2	0.01		$(55)_2$:98	
						$(55)_2$:	$\nu\nu 633\uparrow + 521\uparrow$	99
0_5^+				2.3	0.2		$(20)_5$:95	
					$\widetilde{S}(t,p)=10^{-3}$	$(20)_5$:	$\nu\nu 642\uparrow - 642\uparrow$	58
							$\pi\pi 413\downarrow - 413\downarrow$	22
3_1^+				2.15	3.0		$(43)_1$:88; $(43)_2$: 7	
						$(43)_1$:	$\nu\nu 523\downarrow + 521\downarrow$	32
							$\nu\nu 512\uparrow + 521\downarrow$	11
3_2^+				2.3	1.3		$(43)_2$:90; $(43)_1$: 8	
						$(43)_2$:	$\nu\nu 523\downarrow + 521\downarrow$	68
2_3^-				2.3	0.2		$(32)_3$:97	
						$(32)_3$:	$\pi\pi 523\uparrow - 411\uparrow$	52
							$\nu\nu 633\uparrow - 521\uparrow$	45
1_5^+	2.670			2.7	0.2		$(21)_5$:93	
						$(21)_5$:	$\nu\nu 642\uparrow - 651\uparrow$	81

ph interactions. The condition for elimination of the 0^+ ghost states due to the conservation of the average number of neutrons and protons was used to obtain the equations for monopole and quadrupole pairing. The quadrupole pp interaction is important, because as the constant G^{20} increases the energies of the low-lying poles of the secular equation decrease, and the values of $B(\mathrm{E2})\uparrow$ also decrease. In the calculations with $G^{20}=\kappa_0^{20}$ the calculated values of $B(\mathrm{E2})$ approach the experimental values, the density of low-lying 0^+ states grows, and their structure is changed compared with the case of the calculations for $G^{20}=0$. The wave functions of the low-lying 0^+ states calculated in the RPA are very complicated. They consist of a large number of two-quasiparticle configurations even when the value of $B(\mathrm{E2})$ for a transition to the ground-state rotational band is very small. The 0^+ states are a mixture of pair and quadrupole vibrations.

Excited 0^+ states play a special role in nuclear theory, because their description involves all the mathematical difficulties which can arise. Therefore, the energies and structures of 0^+ states are not described as well as those of other nonrotational states. For example, the calculated energy of the second 0_2^+ state in ^{156}Gd is 0.632 MeV higher than the experimental value. The calculated value of $\widetilde{S}(p,t)$ for excitation of the second 0_2^+ state in the (p,t) reaction in ^{158}Gd is significantly smaller than the experimental value, and so on. We note that the microscopic calculations of $\widetilde{S}(p,t)$ and

TABLE VII. E1 and M1 transitions to the ground state in ^{158}Gd.

Initial state		E1 or $M1$	$B(\mathrm{E1};1^-K_n\to0^+0_{\mathrm{g.s.}})$, $e^2\mathrm{F}^2$ or $B(\mathrm{M1};1^+1_n\to0^+0_{\mathrm{g.s.}})$, μ_N^2	
$I^\pi K_n$	E_n, MeV		experiment (Ref. 36)	calculation
1^-0_1	1.224	E1	$(6.4\pm1.8)\cdot10^{-3}$	$25\cdot10^{-3}$
1^-1_1	1.5*	E1	-	$7\cdot10^{-3}$
1^+1_1	1.9*	M1	-	$1\cdot10^{-3}$
1^-0_2	1.967	E1	$(1.2\pm0.2)\cdot10^{-3}$	$11\cdot10^{-3}$
1^-1_2	1.997	E1	-	$0.9\cdot10^{-3}$
1^+1_2	2.0*	M1	-	$3\cdot10^{-3}$
1^+1_3	2.4*	M1	-	0.02
1^+1_4	2.348	M1	0.07 ± 0.01	0.2
1^-1_3	2.2*	E1	-	$0.3\cdot10^{-3}$
1^+1_5	2.670	M1	0.06 ± 0.01	0.25
1^-1	3.415	E1	$(1.3\pm0.2)\cdot10^{-3}$	$5\cdot10^{-3}$
1^-1	3.460	E1	$(1.1\pm0.2)\cdot10^{-3}$	$2\cdot10^{-3}$
$\left.\begin{matrix}1^-0\\1^-1\end{matrix}\right\}$	2.471	E1	$(1.0\pm0.2)\cdot10^{-3}$	$3\cdot10^{-3}$

*Calculated energies.

$\widetilde{S}(t,p)$ in Ref. 82 correctly reproduce the variation of these quantities in going from one nucleus to another.

In several nuclei, for example, ^{160}Gd, ^{162,164}Dy, and ^{168}Er, the calculated values of $B(\mathrm{E2})$ for excited states with $I^\pi K_n=2^+0_1$ are very small, so that they are not excited experimentally. The calculated values of $B(\mathrm{E2})$ for the excitation of 2^+0_1 states in ^{156,158}Gd, ^{160}Dy, and ^{166}Er are fairly large and agree with the experimental data.

In some cases the values of $B(\mathrm{E2})$ for transitions to 2^+2_1 states are larger than those for transitions to $2^+0_{\mathrm{g.s.}}$. This excess occurs for transitions from the first 0_1^+ and second 0_2^+ states in ^{162,164}Dy and ^{168}Er. It is associated with a very small value of $B(\mathrm{E2})$ for the transition to the $2^+0_{\mathrm{g.s.}}$ state and a 2–4% admixture of the doubly gamma-vibrational configuration in the wave functions of 0^+ states. This excess of the reduced probability for the E2 transition from the first 0_1^+ state to the gamma-vibrational state over that for the transition to the ground-state rotational band is not present for ^{156,158}Gd, ^{160}Dy, and ^{166}Er.

A new interpretation of the first 0_1^+ state as a phonon excitation on a gamma-vibrational state was proposed in Refs. 83 and 84. It is based on dominance of the reduced probability of the E2 transition to the gamma-vibrational state over the same for the transition to the $2^+0_{\mathrm{g.s.}}$ ground state. This interpretation sharply contradicts the QPM calculations. According to our calculations, the contribution of the doubly gamma-vibrational component to the wave function of the 0_1^+ state cannot be greater than 10%. This is sufficient for such dominance if $B(\mathrm{E2};0^+0_1\to2^+0_{\mathrm{g.s.}})$ is very small. As was shown in Ref. 85, the large set of experimental data on one- and two-nucleon transfer reactions and on the E0-transition strengths is inconsistent with the interpretation of the 0_1^+ state proposed in Ref. 83.

The following ratio was studied in Refs. 86 and 87:

$$R_{\beta\gamma}=\frac{E_{0_1^+}}{E_{2_\gamma^+}-E_{2_{\mathrm{g.s.}}^+}},$$

where $E_{2_\gamma^+}$ and $E_{2_{\mathrm{g.s.}}^+}$ are the energies of the gamma-vibrational and the $I^\pi K=2^+0_{\mathrm{g.s.}}$ states. According to Ref. 86, if the interpretation of the 0_1^+ state given in Ref. 83 is correct, the ratio $R_{\beta\gamma}$ should take values between 1.2 and 1.8. The experimental data of Ref. 88 were used in Ref. 89 to show that of the 50 nuclei in the region $150\leqslant A\leqslant190$ the values of $R_{\beta\gamma}$ lie in the range 1.2–1.8 only in 20 cases. For all the nuclei in this region $R_{\beta\gamma}$ takes values from 0.7 to 2.4. In Ref. 89 the calculated values of the energies $E_{0_1^+}$, $E_{2_\gamma^+}$, and $E_{2_{\mathrm{g.s.}}^+}$ were taken from Ref. 76, and very good agreement with the values of $R_{\beta\gamma}$ for the experimental data was obtained. This implies that the ratio $R_{\beta\gamma}$ can be described correctly by various models, and that it may not indicate a two-phonon structure of the first 0_1^+ state.

5.3. States with $K^\pi=1^+$

Low-lying $K^\pi=1^+$ states have been discovered experimentally in a number of even–even deformed nuclei in one-nucleon transfer reactions and in β decay. According to the method we use to exclude the 1^+ ghost state, the first excited $K^\pi=1^+$ state must lie above the first pole, i.e., there must not be any 1^+ states of energy less than 1.5 MeV. Of the nuclei analyzed here, the 1_1^+ state in ^{162}Dy has the lowest energy, equal to 1.746 MeV. In the other nuclei this energy is greater than 1.8 MeV. The values of $B(\mathrm{E2};0^+0_{\mathrm{g.s.}}\to2^+1_n)$ are considerably smaller than those of $B(\mathrm{E2};0^+0_{\mathrm{g.s.}}\to2^+2_1)$. As a rule, they are less than 0.5 single-particle units. In ^{156}Gd and ^{164}Dy the values of $B(\mathrm{E2})$ for excitation of the second 1_2^+ state are larger than for the first 1_1^+ state. The energies and structure of the low-lying $K^\pi=1^+$ states are described fairly well in the QPM.

Collective 1^+ states, which are strongly excited in M1 transitions, lie above 2.5 MeV. The fragmentation of one-phonon states with $K^\pi=1^+$ in the energy range 2.5–4.0 MeV is correctly described in the QPM.[90] Fast M1 transitions of energy of about 2.5 MeV between excited states should be observed in deformed nuclei. They might indicate the presence of large two-phonon components in the wave functions of excited states.[91] Several such fast M1 transitions are seen in Tables IX and XIII. The intensities of M1 transitions are significantly larger than those of E2 transitions between the same one-phonon states.

5.4. States with $K^\pi=2^+$

The first $K_n^\pi=2_1^+$ states in all deformed nuclei are collective states, the so-called gamma-vibrational states. As a rule, their energies are less than 1.4 MeV, and $B(\mathrm{E2};0^+0_{\mathrm{g.s.}}\to2^+2_1)>3$ single-particle units. The energies and the largest two-quasiparticle components of the wave functions of the first 2_1^+ states are correctly described in the QPM. The next three or four $K_n^\pi=2^+$ states below 2.3 MeV are weakly collective one-phonon states with values of $B(\mathrm{E2};0^+0_{\mathrm{g.s.}}\to2^+2_n)$ less than 0.2 single-particle units.

5.5. States with $K^\pi=0^-$ and 1^-

The energies and wave functions of one-phonon states with $K^\pi=0^-$ and 1^- are mainly determined by octupole–

TABLE VIII. Nonrotational states in ^{160}Dy.

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
2_1^+	0.966	4.7		1.0	5.0		$(22)_1$:98	
						$(22)_1$:	$\pi\pi 411\uparrow+411\downarrow$	24
							$\nu\nu 521\uparrow+521\downarrow$	14
							$\nu\nu 642\uparrow-660\uparrow$	9
							$\nu\nu 523\downarrow-521\downarrow$	9
2_1^-	1.265	11		1.3	7.0		$(32)_1$:99	
						$(32)_1$:	$\pi\pi 523\uparrow-411\uparrow$	62
							$\nu\nu 633\uparrow-521\uparrow$	10
0_1^+	1.280	0.71		1.2	0.6		$(20)_1$:97	
							$\{(20)_1,(20)_1\}$: 1	
			$\tilde{S}(p,t)=0.16$		$\tilde{S}(p,t)=0.18$	$(20)_1$:	$\nu\nu 523\downarrow-523\downarrow$	31
			$\tilde{S}(t,p)\leq 0.01$		$\tilde{S}(t,p)=0.32$		$\nu\nu 521\downarrow-521\downarrow$	23
							$\nu\nu 505\uparrow-505\uparrow$	13
							$\pi\pi 411\uparrow-411\uparrow$	12
							$\pi\pi 402\downarrow-402\downarrow$	10
1_1^-	1.285	5.9		1.3	5.0		$(31)_1$:98	
		(d,t): $\nu\nu 642\uparrow-521\uparrow$				$(31)_1$:	$\nu\nu 642\uparrow-521\uparrow$	69
			large				$\nu\nu 523\downarrow-651\uparrow$	5
0_2^+	1.444			1.6	0.001		$(20)_2$:98	
			$\tilde{S}(t,p)=0.02$		$\tilde{S}(t,p)=0.01$	$(20)_2$:	$\nu\nu 521\uparrow-521\uparrow$	58
					$\tilde{S}(p,t)=0.01$		$\nu\nu 523\downarrow-523\downarrow$	41
0_1^-	1.489	6.0		1.5	6.0		$(30)_1$:99	
						$(30)_1$:	$\nu\nu 642\uparrow-523\downarrow$	19
							$\nu\nu 651\uparrow-521\uparrow$	8
4_1^+	1.694			1.7	0.2		$(44)_1$:97	
							$\{(22)_1,(22)_1\}$: 2.3	
		$\log ft=4.69$ from ^{160}Ho:				$(44)_1$:	$\nu\nu 523\downarrow+521\uparrow$	90
			$\nu\nu 523\downarrow+521\uparrow$				$\nu\nu 642\uparrow+651\uparrow$	4
			large					
0_3^+	1.709			1.8	0.2		$(20)_3$:84;$(20)_4$:4	
							$\{(22)_1,(22)_1\}$: 9	
			$\tilde{S}(t,p)=0.05$		$\tilde{S}(t,p)=0.01$	$(20)_3$:	$\pi\pi 411\uparrow-411\uparrow$	52
					$\tilde{S}(p,t)=0.01$		$\nu\nu 505\uparrow-505\uparrow$	12
							$\nu\nu 402\uparrow-402\uparrow$	7
							$\nu\nu 521\uparrow-521\uparrow$	6
4_1^-	1.786		(^{3}He,α):	1.7	0.5		$(54)_1$:98	
			$\nu\nu 642\uparrow+521\uparrow$			$(54)_1$:	$\nu\nu 642\uparrow+521\uparrow$	80
			large				$\pi\pi 523\uparrow+411\downarrow$	12
1_1^+	1.805			1.8	0.001		$(21)_1$: 99	
						$(21)_1$:	$\nu\nu 523\downarrow-521\uparrow$	99
0_4^+	1.953			2.0	0.001		$(20)_4$:88;$(20)_3$:6	
					$\tilde{S}(t,p)=0.04$	$(20)_4$:	$\nu\nu 642\uparrow-642\uparrow$	61
					$\tilde{S}(p,t)=0.05$		$\nu\nu 523\downarrow-523\downarrow$	11
							$\nu\nu 521\uparrow-521\uparrow$	9
							$\pi\pi 411\uparrow-411\uparrow$	4
1_2^-				1.9	2.5		$(31)_2$:93	
						$(31)_2$:	$\nu\nu 523\downarrow-651\uparrow$	43
							$\nu\nu 521\uparrow-642\uparrow$	26
							$\pi\pi 523\uparrow-413\downarrow$	4
4_2^+	2.097			2.04	0.1		$(44)_2$:89	
							$\{(22)_1,(22)_1\}$: 6.2	
							$\{(20)_1,(44)_2\}$: 3	
		$\log ft=6.78$ from ^{160}Ho:				$(44)_2$:	$\nu\nu 642\uparrow+651\uparrow$	90
			$\nu\nu 523\downarrow+521\uparrow$				$\nu\nu 523\downarrow+521\uparrow$	8
			small					
			(d,t)					
0_2^-				2.0	0.6		$(30)_2$:98	
						$(30)_2$:	$\nu\nu 521\uparrow-651\uparrow$	24
							$\nu\nu 523\downarrow-642\uparrow$	22
4_2^-				2.0	1.1		$(54)_2$:98	
						$(54)_2$:	$\pi\pi 523\uparrow+411\downarrow$	47
							$\nu\nu 523\downarrow+651\uparrow$	25
							$\nu\nu 521\uparrow+642\uparrow$	18
5_1^-				2.0	0.03		$(55)_1$:99	

TABLE VIII. (*Continued.*)

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(\mathrm{E}\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(\mathrm{E}\lambda)\uparrow$ one-particle units		Structure	%
						$(55)_1$:	$\nu\nu523\downarrow+642\uparrow$	94
							$\pi\pi523\uparrow+411\uparrow$	5
5_2^-				2.1	0.05		$(55)_2$:93	
						$(55)_2$:	$\pi\pi523\uparrow+411\uparrow$	93
							$\nu\nu523\downarrow+642\uparrow$	5
0_5^+				2.2	0.02		$(20)_5$:89; $(20)_4$: 4	
							$\{(32)_1,(32)_1\}$: 4	
					$\tilde{S}(t,p)=0.001$	$(20)_5$:	$\nu\nu642\uparrow-642\uparrow$	90
					$\tilde{S}(p,t)=0.007$		$\pi\pi411\downarrow-411\downarrow$	6
2_2^+				2.2	0.1		$(22)_2$:72; $(22)_3$:18	
							$\{(20)_1,(22)_1\}$: 3	
							$\{(22)_1,(44)_2\}$: 2	
						$(22)_2$:	$\pi\pi411\uparrow+411\downarrow$	66
							$\nu\nu660\uparrow-642\uparrow$	15
							$\nu\nu521\uparrow+521\downarrow$	7
1_3^-				2.2	0.5		$(31)_3$:95; $(31)_4$: 3	
						$(31)_3$:	$\pi\pi523\uparrow-413\downarrow$	55
							$\nu\nu523\downarrow-651\uparrow$	33
3_1^-				2.2	0.2		$(33)_1$:98	
						$(33)_1$:	$\nu\nu521\uparrow+651\uparrow$	97
4_3^-				2.3	0.2		$(54)_3$:93	
							$\{(22)_1,(32)_1\}$: 5	
						$(54)_3$:	$\nu\nu523\downarrow+651\uparrow$	71
							$\pi\pi523\uparrow+411\downarrow$	28
1_2^+				2.3	0.15		$(21)_2$:94	
							$\{(20)_1,(21)_2\}$: 4	
						$(21)_2$:	$\nu\nu642\uparrow+651\uparrow$	88
							$\nu\nu633\uparrow-642\uparrow$	5
2_2^-				2.3	0.3		$(32)_2$:94	
						$(32)_2$:	$\nu\nu633\uparrow-521\uparrow$	63
							$\pi\pi523\uparrow-411\uparrow$	26
3_3^+	2.524			2.7	0.01		$(43)_3$: 93	
			(d,t) large			$(43)_3$:	$\nu\nu642\uparrow+400\uparrow$	96

TABLE IX. E1 and M1 transitions to the ground state and E1, E2, and M1 transitions between excited states in ^{160}Dy.

Initial state		Eλ or Mλ	Final state		$B(\mathrm{E}\lambda)$, $e^2\mathrm{F}^{2\lambda}$ or $B(\mathrm{M}\lambda)$, $\mu_N^2\mathrm{F}^{2\lambda-2}$			γ-transition probability $\sec^{-1}$
$I^\pi K_n$	E_n, MeV		$I^\pi K_n$	E_n, MeV	exp.	[Ref.]		calculation
2^-2_1	1.265	E1	2^+2_1	0.966	$\geqslant 9\cdot10^{-4}$	48	$14\cdot10^{-4}$	$6\cdot10^{10}$
0^+0_1	1.280	E2	2^+2_1	0.966	-		5.1	-
1^-1_1	1.285	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$9\cdot10^{-3}$	$2\cdot10^{13}$
0^+0_2	1.444	E2	2^+2_1	0.966	-		12.8	-
1^-0_1	1.489	E1	$0^+0_{\mathrm{g.s.}}$	0	$7.2\cdot10^{-3}$	49	$52\cdot10^{-3}$	$3\cdot10^{14}$
4^+4_1	1.694	E2	2^+2_1	0.966	8.8	48	22	$6\cdot10^{9}$
0^+0_3	1.709	E2	2^+2_1	0.966	-		212	-
1^+1_1	1.805	M1	$0^+0_{\mathrm{g.s.}}$	0	-		0.003	$1\cdot10^{11}$
0^+0_4	1.953	E2	2^+2_1	0.966	-		1.0	-
4^+4_2	2.097	E2	2^+2_1	0.966	-		51	10^{11}
1^+1_2	2.3*	M1	$0^+0_{\mathrm{g.s.}}$	0	-		0.30	$6\cdot10^{13}$
1^+1_3	2.4*	M1	$0^+0_{\mathrm{g.s.}}$	0	-		0.03	$2\cdot10^{13}$
1^-0_3	2.4*	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$20\cdot10^{-3}$	$6\cdot10^{15}$
1^-1_5	2.6*	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$1\cdot10^{-3}$	$3\cdot10^{13}$
1^-1_7	2.8*	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$2\cdot10^{-3}$	$6\cdot10^{13}$
1^-0_4	2.9*	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$8\cdot10^{-3}$	$3\cdot10^{15}$
1^-1_8	2.9*	E1	$0^+0_{\mathrm{g.s.}}$	0	-		$3.6\cdot10^{-3}$	10^{14}
2^+2_6	3.0*	E2	4^+4_1	1.694	-		530	$2\cdot10^{12}$
1^-1_{11}	3.1*	E1	2^+2_1	0.966	-		$21\cdot10^{-3}$	$3\cdot10^{14}$
1^-1_{12}	3.2*	M1	2^-2_1	1.265	-		$4\cdot10^{-3}$	$5\cdot10^{11}$
1^+1_{24}	3.7*	M1	2^+2_1	0.966	-		0.072	$3\cdot10^{13}$

*Calculated energies.

TABLE X. Nonrotational states in ^{162}Dy.

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure		%
2_1^+	0.888	4.5		0.9	5.0		$(22)_1$:98	
			(d,p)			$(22)_1$:	$\nu\nu521\uparrow+521\downarrow$	24
							$\pi\pi411\uparrow+411\downarrow$	17
							$\nu\nu523\downarrow-521\downarrow$	14
							$\nu\nu642\uparrow-660\uparrow$	6
							$\pi\pi413\downarrow-411\downarrow$	5
2_1^-	1.148	9.6		1.2	7.0		$(32)_1$:98	
		$\log ft=4.95$ from ^{162}Tb:				$(32)_1$:	$\pi\pi523\uparrow-411\uparrow$	52
		$\pi\pi523\uparrow-411\uparrow$ large					$\nu\nu633\uparrow-521\uparrow$	17
		$(d,p)(d,t)$					$\nu\nu642\uparrow-521\downarrow$	5
0_1^-	1.275	4.7		1.3	5.5		$(30)_1$:99	
		(^{3}He,α)				$(30)_1$:	$\nu\nu523\downarrow-642\uparrow$	34
			$\nu\nu523\downarrow-642\uparrow$				$\nu\nu521\downarrow-651\uparrow$	2
		(α,^{3}He)	large					
		$(d,p),(d,t)$						
0_1^+	1.398	$\tilde{S}(t,p)=0.03$		1.4	0.2		$(20)_1$: 97	
							$\{(22)_1,(22)_1\}$:2	
		$\log ft=5.1$ from ^{162}Ho:				$(20)_1$:	$\nu\nu523\downarrow-523\downarrow$	50
		$\nu\nu523\uparrow-523\uparrow$ large					$\nu\nu642\uparrow-642\uparrow$	23
		(α,^{3}He)	$\nu\nu642\uparrow-642\uparrow$				$\pi\pi411\uparrow-411\uparrow$	7
		$(d,p),(d,t)$	large			$\tilde{S}(t,p)=0.04$,	$\tilde{S}(p,t)=0.01$	
5_1^-	1.486	$\log ft=4.5$ from ^{162}Ho:		1.5	0.01		$(55)_1$: 99	
		$\nu\nu523\downarrow+642\uparrow$ large				551:	$\nu\nu523\downarrow+642\uparrow$	98
		(^{3}He,α)						
		(α,^{3}He)	$\nu\nu523\uparrow+642\uparrow$					
		(d,t)	large					
		(d,p)						
4_1^+	1.536	(^{3}He,α)	$\nu\nu523\downarrow+521\uparrow$	1.5	1.2		$(44)_1$: 97	
							$\{(22)_1,(22)_1\}$: 2.3	
		(d,t)	large			$(44)_1$:	$\nu\nu523\downarrow+521\uparrow$	70
			(d,p)				$\pi\pi413\downarrow+411\uparrow$	10
							$\nu\nu642\uparrow+651\uparrow$	7
3_1^-	1.571	$(d,p)\}$	$\nu\nu642\uparrow+521\downarrow$	1.5	4.3		$(33)_1$: 97	
			large			$(33)_1$:	$\nu\nu642\uparrow+521\downarrow$	48
							$\pi\pi514\uparrow-411\uparrow$	17
							$\pi\pi523\uparrow-411\downarrow$	2
1_1^-	1.637		(d,p)	1.6	2.8		$(31)_1$: 99	
						$(31)_1$:	$\nu\nu642\uparrow-521\uparrow$	68
							$\nu\nu633\uparrow-523\downarrow$	8
0_2^+	1.666	$\tilde{S}(t,p)\leq0.004$		1.7	0.07		$(20)_2$: 89; $(20)_3$:9	
						$(20)_2$:	$\nu\nu521\uparrow-521\uparrow$	80
							$\pi\pi411\uparrow-411\uparrow$	4
		$\tilde{S}(p,t)=0.13$					$\nu\nu523\downarrow-523\downarrow$	3
						$\tilde{S}(t,p)=0.03$	$\tilde{S}(p,t)=0.05$	
1_1^+	1.746	(d,t)	$\nu\nu523\downarrow-521\uparrow$	1.8	$7\cdot10^{-4}$		$(21)_1$: 99	
			large			$(21)_1$:	$\nu\nu523\downarrow-521\uparrow$	99
	1.840	for $I^\pi K_\nu=3^+1_1$						
		(^{3}He,α)	$\nu\nu523\downarrow+521\uparrow$					
		(d,t)	large					
3_2^-	1.767	(α,^{3}He)	$\nu\nu642\uparrow+521\downarrow$	1.8	3.2		$(33)_2$: 96	
		(d,p)	large				$\{(22)_1,(55)_1\}$: 1	
							$\{(20)_1,(33)_1\}$: 1	
						$(33)_2$:	$\nu\nu642\uparrow+521\downarrow$	52
							$\pi\pi514\uparrow-411\uparrow$	17
							$\pi\pi523\uparrow-411\downarrow$	3
6_1^-	(1.807)	$(d,p)\}$	$\nu\nu523\downarrow+633\uparrow$	1.9			$\nu\nu523\downarrow+633\uparrow$	100
4_1^-				1.9	1.6		$(54)_1$: 96	
							$\{(22)_1,(32)_1\}$: 3	
						$(54)_1$:	$\pi\pi523\uparrow+411\downarrow$	45
							$\nu\nu523\downarrow+651\uparrow$	18
							$\nu\nu521\uparrow+642\uparrow$	15
0_3^+	2.127	$(d,p)\}$	$\nu\nu642\uparrow-642\uparrow$	1.9	0.01		$(20)_3$: 88; $(20)_2$: 9	
						$(20)_3$:	$\nu\nu521\downarrow-521\downarrow$	34
		$\tilde{S}(t,p)=0.08$					$\nu\nu633\uparrow-633\uparrow$	30

TABLE X. (*Continued.*)

K_n^π	Experiment E_n MeV	Experiment $B(E\lambda)\uparrow$ one-particle units	Experiment Structure	QPM calculations E_n MeV	QPM calculations $B(E\lambda)\uparrow$ one-particle units	QPM calculations Structure		%
							$\nu\nu 642\uparrow-642\uparrow$	24
						$\tilde{S}(t,p)=0.2$	$\tilde{S}(p,t)=0.01$	
2_2^-	1.864	$(d,p)\}$	$\nu\nu 642\uparrow+521\downarrow$	2.0	0.02		$(32)_2$: 99	
			large			$(32)_2$:	$\nu\nu 642\uparrow-521\downarrow$	94
							$\pi\pi 523\uparrow-411\uparrow$	5
0_2^-				2.0	3.0		$(30)_2$: 98	
						$(30)_2$:	$\nu\nu 523\downarrow-642\uparrow$	18
							$\nu\nu 521\uparrow-651\uparrow$	12
2_2^+	(1.999)	$(d,p)\}$	$\nu\nu 523\downarrow+521\downarrow$	2.1	0.2		$(22)_2$: 88; $(22)_4$: 2	
							$\{(20)_1,(22)_1\}$: 2	
							$\{(22)_1,(44)_1\}$: 6	
						$(22)_2$:	$\nu\nu 521\uparrow+521\downarrow$	69
							$\pi\pi 411\uparrow+411\downarrow$	19
							$\nu\nu 523\downarrow-521\downarrow$	8
1_2^-				2.0	0.2		$(31)_2$: 99	
						$(31)_2$:	$\nu\nu 633\uparrow-523\downarrow$	82
							$\nu\nu 642\uparrow-521\uparrow$	12
3_1^+	2.283	for $I^\pi K_\nu=5^+3_1$		2.1	1.3		$(43)_1$: 98	
		$(^3\text{He},\alpha)\}$	$\nu\nu 505\uparrow-523\downarrow$			$(43)_1$:	$\nu\nu 505\uparrow-523\downarrow$	78
			large					
5_2^-				2.1	0.1		$(55)_2$: 98	
						$(55)_2$:	$\pi\pi 523\uparrow+411\uparrow$	99
0_4^+				2.1	0.06		$(20)_4$: 95	
							$\{(32)_1,(32)_1\}$: 1	
						$(20)_4$:	$\pi\pi 411\uparrow-411\uparrow$	59
							$\nu\nu 523\downarrow-523\downarrow$	21
							$\pi\pi 411\downarrow-411\downarrow$	6
							$\nu\nu 642\uparrow-642\uparrow$	5
						$\tilde{S}(t,p)=0.001$	$\tilde{S}(p,t)=0.003$	
1_2^+	2.623			2.3	0.3		$(21)_2$: 95	
		for $I^\pi K_\nu=6^+1_2$				$(21)_2$:	$\nu\nu 633\uparrow-642\uparrow$	55
		$(\alpha,^3\text{He})\}$	$\nu\nu 633\uparrow-642\uparrow$				$\nu\nu 642\uparrow-651\uparrow$	18
			large				$\pi\pi 532\uparrow-523\uparrow$	17
4_2^+				2.2	0.3		$(44)_2$: 78; 443: 4	
							$\{(22)_1,(22)_1\}$: 16	
						$(44)_2$:	$\nu\nu 642\uparrow+651\uparrow$	59
							$\pi\pi 413\downarrow+411\uparrow$	23
							$\nu\nu 523\downarrow+521\uparrow$	16
1_3^-				2.2	1.9		$(31)_3$: 98	
						$(31)_3$:	$\nu\nu 651\uparrow-523\downarrow$	30
							$\nu\nu 642\uparrow-521\uparrow$	18
							$\pi\pi 523\uparrow-413\downarrow$	14
4_2^-				2.2	0.2		$(54)_2$: 93	
							$\{(22)_1,(32)_1\}$: 3	
						$(54)_2$:	$\nu\nu 521\uparrow+642\uparrow$	84
							$\nu\nu 523\uparrow+651\uparrow$	9
							$\pi\pi 523\uparrow+411\downarrow$	6
0_5^+				2.2	0.02		$(20)_5$: 95; 206: 3	
						$(20)_5$:	$\nu\nu 633\uparrow-633\uparrow$	46
							$\nu\nu 521\downarrow-521\downarrow$	33
							$\nu\nu 523\downarrow-521\uparrow$	10
2_3^+				2.2	0.2		$(22)_3$: 93; $(22)_1$: 1	
							$\{(20)_1,(22)_1\}$: 3	
						$(22)_3$:	$\nu\nu 523\downarrow-521\downarrow$	56
							$\pi\pi 411\uparrow+411\downarrow$	42
3_2^+				2.3	0.1		$(43)_2$: 98	
						$(43)_2$:	$\nu\nu 523\downarrow+521\downarrow$	96
4_3^-				2.3	0.1		$(54)_3$: 89; $(54)_2$: 4	
							$\{(22)_1,(32)_1\}$: 6	
						$(54)_3$:	$\nu\nu 523\downarrow+651\uparrow$	67
							$\pi\pi 523\uparrow+411\downarrow$	20
8_1^+	2.203	$(^3\text{He},\alpha)\}$	$\nu\nu 505\uparrow+523\downarrow$	2.5			$\nu\nu 505\uparrow+523\downarrow$	100
			large					
2_3^-	2.371	log ft=5.33 from ^{162}Tb:		2.3	0.1		$(32)_3$: 98	

TABLE X. (*Continued.*)

K_n^π	Experiment E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	QPM calculations E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure		%
			$\pi\pi523\uparrow-411\uparrow$			$(32)_3$:	$\nu\nu633\uparrow-521\uparrow$	70
			significant				$\pi\pi523\uparrow-411\uparrow$	28
6_1^+	2.505			2.3			$\nu\nu642\uparrow+633\uparrow$	100
		for $I^\pi K_\nu=7^+6_1$						
		$(\alpha,{}^3\mathrm{He})\}$	$\nu\nu642\uparrow+633\uparrow$					
			large					
2_4^+				2.3	0.004		$(22)_4$: 83	
							$\{(30)_1,(32)_1\}$: 3	
							$\{(22)_1,(44)_1\}$: 2	
						$(22)_4$:	$\nu\nu642\uparrow-660\uparrow$	73
							$\nu\nu523\downarrow-521\downarrow$	12
							$\pi\pi411\uparrow+411\downarrow$	6
5_3^-				2,3	0.01		$(55)_3$: 98	
						$(55)_3$:	$\nu\nu521\uparrow+633\uparrow$	99
1_4^-				2.4	0.02		$(31)_4$: 98	
						$(31)_4$:	$\pi\pi523\uparrow-413\downarrow$	77
							$\nu\nu651\uparrow-523\downarrow$	22
3_3^+				2.4	2.5		$(43)_3$: 94	
							$\{(30)_1,(33)_1\}$: 1	
						$(43)_3$:	$\nu\nu505\uparrow-523\downarrow$	19
							$\nu\nu512\uparrow+521\uparrow$	16
							$\nu\nu633\uparrow-660\uparrow$	12
4_3^+				2.4	0.2		$(44)_3$: 35; $(44)_2$: 17	
							$\{(22)_1,(22)_1\}$: 40	
						$(44)_3$:	$\pi\pi413\downarrow+411\uparrow$	62
							$\nu\nu642\uparrow+651\uparrow$	24
3_3^-				2.4	0.01		$(33)_3$: 99	
						$(33)_3$:	$\nu\nu521\uparrow+651\uparrow$	99
1_3^+				2.4	0.12		$(21)_3$: 97	
							$\{(22)_1,(43)_4\}$: 2	
						$(21)_3$:	$\nu\nu642\uparrow-651\uparrow$	70
							$\nu\nu633\uparrow-642\uparrow$	16
0_3^-	2.520			2.4	0.6		$(30)_3$: 96	
							$\{(30)_1,(20)_2\}$: 1	
						$(30)_3$:	$\nu\nu521\uparrow-651\uparrow$	33
							$\nu\nu512\uparrow-642\uparrow$	15

octupole interactions. The first $K_n^\pi=0_1^-$ and 1_1^- states are collective states with values of $B(\mathrm{E3};0^+0_{\mathrm{g.s.}}\rightarrow 3^-K_n)$ lying in the range of 2–12 single-particle units. Their energies and $B(\mathrm{E3})$ values vary considerably in going from nucleus to nucleus. Their collective nature is weakened and their energies increase in going from Gd to Er isotopes. The experimental data on the second and third 0_2^-, 0_3^-, 1_2^-, and 1_3^- states is extremely sparse.

If one-phonon states with $K^\pi=0^-$ and 1^- are described with inclusion of octupole–octupole ph and pp interactions, the calculated values of $B(\mathrm{E1};0^+0_{\mathrm{g.s.}}\rightarrow 1^-0_i)$ and $B(\mathrm{E1};0^+0_{\mathrm{g.s.}}\rightarrow 1^-1_i)$ are two orders of magnitude greater than the experimental values. As was shown in Ref. 25, inclusion of the isovector dipole–dipole ph interaction with the constant $\kappa_1^{1K}=-1.5\kappa_0^{3K}$, at which the location of the isovector giant dipole resonance is correctly described, causes the values of $B(\mathrm{E1})$ to decrease by roughly a factor of 20. Comparison with the experimental data[49] has shown that the calculated values of $B(\mathrm{E1};0^+0_{\mathrm{g.s.}}\rightarrow 1^-0_i)$ and $B(\mathrm{E1};0^+0_{\mathrm{g.s.}}\rightarrow 1^-1_i)$ are 3–5 times larger than the experimental values.[25] Moreover, the total strength of E1 transitions in the energy range 0–4 MeV with $K^\pi=0^-$ is 3–4 times larger than for states with $K^\pi=1^-$.

The probabilities of E1 transitions between one-phonon terms of the wave functions of the initial and final states depend on their small two-quasiparticle components. Therefore, the results of these calculations are not very reliable. The experimental reduced E1-transition probabilities and the probabilities of decays (per second) between one-phonon terms of the wave functions of the initial and final states are $B(\mathrm{E1})=(10^{-3}-10^{-7})\ e^2\mathrm{F}^2$ and $T(\mathrm{E1})=(10^5-10^{11})\ \mathrm{sec}^{-1}$. Similar small $B(\mathrm{E1})$ values calculated in the QPM are found in Tables III, V, XI, and XVII.

According to the QPM calculations, the intensity of E1 transitions is large if the wave function of the initial state has a relatively large two-phonon term consisting of an octupole phonon with $K^\pi=0^-$ or 1^- and another phonon which is in the wave function of the final state. Examples can be cited (see Ref. 91) in which the intensity of such transitions is $10-10^3$ times larger than the intensity of transitions to the ground states and 10^3-10^6 times larger than the intensities of transitions between one-phonon states.

TABLE XI. E1 and M1 transitions to the ground state and E1, E2, and M1 transitions between excited states in ^{162}Dy.

Initial state				Final state		$B(E\lambda)\downarrow$, e^2 $F^{2\lambda}$ or $B(M\lambda)\downarrow$, μ_N^2 $F^{2\lambda-2}$	
$I^\pi K_n$	E_n, MeV	Eλ or Mλ	n_f	$I^\pi K_n$	E_n, MeV	exp. [Ref.]	calculation
2^-2_1	1.148	E1	1	2^+2_1	0.888	$9.5\cdot10^{-5}$ 50	0.003
		E1	2	3^+2_1	0.963	$4.8\cdot10^{-5}$ 50	0.002
1^-0_1	1.276	E1	1	$0^+0_{g.s.}$	0	$4.9\cdot10^{-3}$ 49	$12\cdot10^{-3}$
2^+0_1	1.453	E2	1	$2^+0_{g.s.}$	0.081	-	10
		E2	2	2^+2_1	0.888	-	16
		E1	3	1^-0_1	1.276	-	$6\cdot10^{-5}$
4^+4_1	1.536	E4	1	$0^+0_{g.s.}$	0	-	$4\cdot10^5$
		E2	2	2^+2_1	0.888	17 51	23
		M2	3	4^-2_1	1.297	-	$8\cdot10^{-5}$
3^-3_1	1.571	E1	1	2^+2_1	0.888	-	10^{-4}
		M1	2	2^-2_1	1.148	-	0.04
1^-1_1	1.637	E1	1	$2^+0_{g.s.}$	0.081	-	$5\cdot10^{-3}$
		M1	2	2^-2_1	1.148	-	10^{-3}
		E2	2	2^-2_1	1.148	-	0.2
		M1	3	1^-0_1	1.276	-	10^{-4}
3^-1_1	1.739	E1	1	$2^+0_{g.s.}$	0	-	$6\cdot10^{-3}$
		E1	2	2^+2_1	0.888	-	$4\cdot10^{-6}$
		M1	3	2^-2_1	1.148	-	$5\cdot10^{-4}$
2^+0_2	1.728	E2	1	$0^+0_{g.s.}$	0	-	1.0
		E2	2	2^+2_1	0.888	-	2.0
		E1	3	1^-0_1	1.276	-	10^{-5}
1^+1_1	1.746	M1	1	2^+2_1	0.888	-	$6\cdot10^{-3}$
		M1	2	2^+0_1	1.453	-	10^{-3}
		M1	3	$0^+0_{g.s.}$	0	-	$3\cdot10^{-4}$
3^-3_2	1.767	E1	1	2^+2_1	0.888	-	10^{-3}
		M1	2	2^-2_1	1.148	-	0.03
		E2	3	5^-5_1	1.486	-	20
		E1	4	4^+4_1	1.536	-	$2\cdot10^{-4}$
3^-2_2	1.910	E1	1	2^+2_1	0.888	-	10^{-5}
		E2	2	3^-0_1	1.358	-	3.0
(4^-2_2)	1.973	E1	1	3^+2_1	0.963	-	10^{-5}
		M1	2	4^-2_1	1.297	-	$7\cdot10^{-3}$
		E2	3	5^-0_1	1.518	-	2.5
		M2	4	4^+0_1	1.574	-	0.1
1^-0_2	1.986	E1	1	$0^+0_{g.s.}$	0	$4\cdot10^{-3}$ 49	$18\cdot10^{-3}$
(2^+2_2)	1.999	E2	1	$0^+0_{g.s.}$	0	-	10
		M1	2	2^+2_1	0.888	-	$2\cdot10^{-3}$
		E1	3	3^-2_1	1.210	-	$2\cdot10^{-5}$
2^-2_3	2.371	E1	1	2^+2_1	0.888	-	$3\cdot10^{-4}$
		M1	2	2^-2_1	1.148	-	0.19
1^-0_3	2.520	E1	1	$0^+0_{g.s.}$	0	$2\cdot10^{-3}$ 52	$5\cdot10^{-3}$

The reduced probabilities of E1 and E3 transitions from the ground state to excited states with $K^\pi=0^-$ and 1^- and between excited states have been calculated in Ref. 92. It was shown that there is a correlation between the reduced probabilities for E1 and E3 transitions from the ground states. According to the calculations, the intensities of E1 transitions are 10^3-10^{10} times larger than the intensities of E3 transitions between the corresponding states. This implies that states whose wave functions have a large two-phonon term containing a phonon with $K^\pi=0^-$ and 1^- can be discovered experimentally from a fast E1 transition.

5.6. Octupole states with $K^\pi=2^-$ and 3^-

The energies and wave functions of one-phonon states with $K^\pi=2^-$ and 3^- are determined by octupole–octupole interactions. The first $K_n^\pi=2_1^-$ states in ^{156,158}Gd are located near 1.8 MeV. In ^{160,162,164}Dy their energies decrease to 1.0–1.3 MeV, and their collective nature is significantly enhanced. In ^{166,168}Er their energy increases to 1.46–1.57 MeV, and their collective nature is weakened. The second $K_n^\pi=2_2^-$ states are located near 2 MeV. The energy and structure of states with $K^\pi=2^-$ are described fairly well by the QPM.

There is little experimental information about states with $K^\pi=3^-$. The first $K_n^\pi=3_1^-$ states in ^{160}Gd, ^{162}Dy, and ^{166}Er are located at the energies 1.452, 1.571, and 1.916 MeV, respectively, and the B(E3) values are unknown. The states with $K^\pi=3^-$ in ^{168}Er, where there are six such states, have unusual behavior.[69] The first three states 3_1^-, 3_2^-, and 3_3^- are weakly collective, and account for 1.3 one-particle

TABLE XII. Nonrotational states in ^{164}Dy.

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure		%
2_1^+	0.762	3.6		0.8	4.1		$(22)_1$:98	
		(d,p): νν523↓−521↓				$(22)_1$:	νν523↓−521↓	35
			large				ππ411↑+411↓	18
							νν521↑+521↓	16
							ππ413↓−411↓	6
2_1^-	0.997	7.9		1.0	5.4		$(32)_1$:99	
		(t,α): ππ523↑−411↑ 47%				$(32)_1$:	ππ 523↑−411↑	70
							νν633↑−521↑	10
4_1^-	1.588	(t,α): ππ523↑+411↓ 20%		1.6	2.4		$(54)_1$:95	
							$\{(22)_1,(32)_1\}$: 3	
						$(54)_1$:	νν633↑+521↓	45
							ππ523↑+411↓	30
0_1^+	1.655			1.6	0.1		$(20)_1$:85; $(20)_2$: 1	
			$\widetilde{S}(t,p)=0.1$		$\widetilde{S}(t,p)=0.1$		$\{(22)_1,(22)_1\}$: 4	
							$\{(32)_1,(32)_1\}$: 3	
						$(20)_1$:	νν521↓−521↓	44
							νν523↓−523↓	13
							νν642↑−642↑	10
							νν512↑−512↑	9
							νν633↑−633↑	8
							ππ411↑−411↑	3
0_1^-	1.675	3.0		1.8	2.0		$(30)_1$:99	
						$(30)_1$:	νν512↑−642↑	36
							νν523↓−642↑	4
6_1^-	1.680	(d,p): νν523↓+633↑		1.7	0.1		$(76)_1$: 99	
			large			$(76)_1$:	νν523↓+633↑	92
							ππ523↑+413↓	3
0_2^+	1.744			1.8	0.04		$(20)_2$:82; $(20)_3$: 9	
			$\widetilde{S}(t,p)=0.2$		$\widetilde{S}(t,p)=0.1$		$\{(22)_1,(22)_1\}$: 2	
						$(20)_2$:	νν633↑−633↑	63
							νν521↓−521↓	30
							νν523↓−523↓	7
2_2^+	(1.796)			1.7	0.4		$(22)_2$:99	
						$(22)_2$:	νν521↑+521↓	45
							ππ411↑+411↓	20
							νν523↓−521↓	20
3_1^-				1.8	0.1		$(33)_1$:99	
						$(33)_1$:	νν633↑−521↓	98
1_1^-	1.809			1.8	2.9		$(31)_1$:98	
						$(31)_1$:	νν633↑−523↓	37
							νν633↑−512↑	22
							νν642↑−521↑	8
0_3^+				1.9	0.07		$(20)_2$:10; $(20)_3$:82	
							$\{(22)_1,(22)_1\}$: 3	
							$\{(32)_1,(32)_1\}$: 3	
						$(20)_3$:	ππ411↑−411↑	51
							ππ523↑−523↑	24
							ππ633↑−633↑	8
1_1^+	1.841		(n,γ)	2.0	0.002		$(21)_1$:98	
						$(21)_1$:	ππ411↑−411↓	94
							νν521↑−521↓	5
2_3^+	(1.921)			2.0	0.1		$(22)_3$:96	
							$\{(32)_1,(54)_1\}$: 2	
						$(22)_3$:	ππ411↑+411↓	45
							νν521↑+521↓	27
							νν523↑−521↓	25
1_2^+	1.948	(≈2)		2.0	0.7		$(21)_2$:97	
			$(n,\gamma)(e,e')$			$(21)_2$:	νν633↑−642↑	60
							νν521↑−521↓	18
							ππ523↑−532↑	9
2_2^-	1.949			1.9	0.1		$(32)_2$:99	
						$(32)_2$:	νν642↑−521↓	80
							νν633↑−521↑	10
							νν523↑−411↑	6
3_1^+	1.979	(d,p): νν523↓+521↓		1.9	0.3		$(43)_1$:90; $(43)_2$: 8	

TABLE XII. (*Continued.*)

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure		%
			large			$(43)_1$:	$\nu\nu523\downarrow+521\downarrow$	62
							$\nu\nu512\downarrow+521\downarrow$	30
0_4^+				2.0	0.1		$(20)_1$: 6; $(20)_2$: 7	
							$(20)_4$: 33; $(20)_5$: 2	
							$(20)_6$: 10	
							$\{(22)_1,(22)_1\}$: 6	
							$\{(32)_1,(32)_1\}$: 35	
						$(20)_4$:	$\nu\nu512\uparrow-512\uparrow$	25
							$\nu\nu523\downarrow-523\downarrow$	20
							$\nu\nu642\uparrow-642\uparrow$	11
							$\pi\pi523\uparrow-523\uparrow$	10
							$\nu\nu521\uparrow-521\uparrow$	6
3_2^-				2.0	0.2		$(33)_2$:97	
						$(33)_2$:	$\nu\nu642\uparrow+521\downarrow$	95
5_1^-	1.988	(t,α): $\pi\pi523\uparrow+411\uparrow$ 42%		2.0	0.05		$(55)_1$: 99	
						$(55)_1$:	$\nu\nu633\uparrow+521\uparrow$	70
							$\pi\pi523\uparrow+411\uparrow$	27
3_2^+	(2.113)			2.0	1.2		$(43)_1$: 9; $(43)_2$: 88	
						$(43)_2$:	$\nu\nu512\uparrow+521\downarrow$	37
							$\nu\nu523\downarrow+521\downarrow$	35
							$\pi\pi514\uparrow-541\uparrow$	8
2_4^+	(2.055)		(d,p)	2.1	0.01		$(22)_4$:97	
						$(22)_4$:	$\nu\nu512\uparrow-521\downarrow$	68
							$\nu\nu633\uparrow-651\uparrow$	26
1_2^-				2.0	0.2		$(31)_2$:97	
						$(31)_2$:	$\nu\nu633\uparrow-523\downarrow$	37
							$\nu\nu633\uparrow-512\uparrow$	25
							$\nu\nu642\uparrow-521\uparrow$	19
4_2^-				2.0	0.01		$(54)_2$:18	
							$\{(22)_1,(32)_1\}$:77	
0_5^+				2.1	$2\cdot10^{-4}$		$(20)_4$:36; $(20)_5$:10	
							$(20)_6$: 4	
							$\{(32)_1,(32)_1\}$:33	
							$\{(22)_1,(22)_1\}$:15	
1_3^+				2.1	0.1		$(21)_3$:98	
						$(21)_3$:	$\nu\nu521\uparrow-521\downarrow$	75
							$\nu\nu633\uparrow-642\uparrow$	14
							$\pi\pi411\uparrow-411\downarrow$	4
2_3^-				2.2	0.5		$(32)_3$:97	
						$(32)_3$:	$\nu\nu633\uparrow-521\uparrow$	67
							$\pi\pi523\uparrow-411\uparrow$	16
4_1^+	2.206			2.1	0.1		$(44)_1$: 2; $(44)_4$: 1	
							$\{(22)_1,(22)_1\}$:95	
4_2^+				2.2	0.5		$(44)_1$:95	
							$\{(22)_1,(22)_1\}$: 3	
						$(44)_1$:	$\nu\nu523\downarrow+521\uparrow$	47
							$\pi\pi413\downarrow+411\uparrow$	30
0_6^+				2.2	0.01		$(20)_3$: 4; $(20)_4$:10	
							$(20)_6$:50	
							$\{(22)_1,(22)_1\}$:22	
							$\{(32)_1,(32)_1\}$:11	
1_3^-				2.2	0.8		$(31)_3$:98	
						$(31)_3$:	$\nu\nu633\uparrow-512\uparrow$	49
							$\nu\nu642\uparrow-521\uparrow$	22
6_2^-				2.2	0.1		$(76)_2$:98	
						$(76)_2$:	$\nu633\uparrow+512\uparrow$	90
							$\pi\pi523\uparrow+413\downarrow$	5
4_3^-				2.2	0.03		$(54)_2$:80	
							$\{(22)_1,(32)_1\}$:17	
						$(54)_2$:	$\nu\nu633\uparrow+521\downarrow$	50
							$\pi\pi523\uparrow+411\downarrow$	40
6_3^-	2.413	(t,α): $\pi\pi523\uparrow+413\downarrow$ 32%		2.4	0.2		$(76)_3$:98	
						$(76)_3$:	$\pi\pi523\uparrow+413\downarrow$	85
							$\nu\nu633\uparrow+512\uparrow$	6

TABLE XIII. E1 and M1 transitions to the ground state and E1, M1, and E2 transitions between excited states in ^{164}Dy.

Initial state		Eλ or $M1$	Final state		$B(\mathrm{E}\lambda)$ (e^2 F$^{2\lambda}$) or $B(\mathrm{M1})$ (μ_N^2)		$W^{\lambda}(i\rightarrow f)$ (sec^{-1})
$I^{\pi}K_n$	E_n MeV		$I^{\pi}K_n$	E_n MeV	exp. [Ref.]	calculation	
2^-2_1	0.997	E1	2^+2_1	0.76		$4\cdot 10^{-3}$	$6\cdot 10^{11}$
4^-4_1	1.588	E2	2^-2_1	1.0		9.9	$1\cdot 10^{9}$
0^+0_1	1.655	E2	2^+2_1	0.76		89	$6\cdot 10^{10}$
1^-0_1	1.675	E1	$0^+0_{\mathrm{g.s.}}$	0	$(7.3\pm 1.0)\cdot 10^{-3}$ 49	$19\cdot 10^{-3}$	$2\cdot 10^{14}$
0^+0_2	1.774	E2	2^+2_1	0.76		27	$4\cdot 10^{10}$
1^-1_1	1.809	E1	$2^+0_{\mathrm{g.s.}}$	0.073		$5\cdot 10^{-3}$	$4\cdot 10^{13}$
		M1	2^-2_1	0.98		0.15	$2\cdot 10^{12}$
1^+1_1	1.841	M1	$0^+0_{\mathrm{g.s.}}$	0		0.08	$1\cdot 10^{13}$
1^+1_2	1.949	M1	$0^+0_{\mathrm{g.s.}}$	0		0.17	$2.6\cdot 10^{-13}$
3^+3_1	1.979	M1	2^+2_1	0.76		$2\cdot 10^{-4}$	$6\cdot 10^{9}$
		E1	2^-2_1	0.98		$6\cdot 10^{-7}$	$1\cdot 10^{11}$
		E1	4^-4_1	1.59		$2\cdot 10^{-5}$	$2\cdot 10^{8}$
3^+3_2	2.113	M1	3^+2_1	0.83		$1.8\cdot 10^{-3}$	$\cdot 10^{10}$
		E1	2^-2_1	0.98		$5\cdot 10^{-5}$	$1\cdot 10^{11}$
4^+4_1	2.1*	E2	2^+2_1	0.76		440	$4\cdot 10^{12}$
1^+1_3	2.1*	M1	$0^+0_{\mathrm{g.s.}}$	0		0.26	$4.3\cdot 10^{13}$
4^+4_2	2.2*	E2	2^+2_1	0.76		7.3	$6\cdot 10^{10}$
1^+1_4	2.3*	M1	$0^+0_{\mathrm{g.s.}}$	0		$2\cdot 10^{-4}$	$5\cdot 10^{10}$
1^-0_2	2.330	E1	$0^+0_{\mathrm{g.s.}}$	0	$(2.0\pm 0.3)\cdot 10^{-3}$ 47	$9\cdot 10^{-3}$	$5\cdot 10^{14}$
1^-0_3	2.671	E1	$0^+0_{\mathrm{g.s.}}$	0	$(1.4\pm 0.2)\cdot 10^{-3}$ 47	$13\cdot 10^{-3}$	$1\cdot 10^{15}$
1^+1_{12}	3.05*	M1	$0^+0_{\mathrm{g.s.}}$	0		$2\cdot 10^{-3}$	$9\cdot 10^{11}$
		M1	2^+2_1	0.76		0.06	$1\cdot 10^{13}$
		E2	2^+2_1	0.76		3.7	$3\cdot 10^{12}$
1^11_{14}	3.10*	M1	$0^+0_{\mathrm{g.s.}}$	0		0.003	$2\cdot 10^{12}$
		M1	2^+2_1	0.76		0.17	$4\cdot 10^{13}$
		E2	2^+2_1	0.76		18	$1.5\cdot 10^{12}$
1^-1_{15}	3.24*	E1	$0^+0_{\mathrm{g.s.}}$	0		$0.2\cdot 10^{-3}$	$1\cdot 10^{13}$
		E1	2^+2_1	0.76		$6\cdot 10^{-3}$	$1.4\cdot 10^{14}$
1^-1_{26}	3.84*	E1	$0^+0_{\mathrm{g.s.}}$	0		$2\cdot 10^{-6}$	$1.2\cdot 10^{11}$
		M1	1^-0_1	1.675		0.016	$3\cdot 10^{12}$
		E1	1^+1_1	1.841		$1.7\cdot 10^{-3}$	$2.2\cdot 10^{13}$

*Calculated energies.

units. The fourth state with $K_n^{\pi}=3_4^-$ is a collective state with $B(\mathrm{E3})=4.68$ one-particle units, i.e., almost three times larger than the first three. The first three states 3_1^-, 3_2^-, and 3_3^- cannot be considered to be two-quasiparticle states, since the $B(\mathrm{E3})$ values for their excitation are 30–60 times larger than the $B(\mathrm{E3})$ values for the corresponding two-quasiparticle states. This unusual distribution of E3 strength among the low-lying states in ^{168}Er is correctly described in Ref. 93 using the QPM. The reason for this unusual distribution is explained in Ref. 15. The matrix elements corresponding to the first three poles of the secular equation are small, and the corresponding roots lie near the poles. The matrix element corresponding to the fourth pole is large. In addition, the fourth pole lies 0.8 MeV above the third. The fourth root is much lower than the fourth pole, so that the state with $K_n^{\pi}=3_4^-$ is a collective state. It was shown in Ref. 15 that a nonstandard E3-strength distribution of this type can occur in other even–even deformed nuclei. On the other hand, there are insurmountable difficulties in describing such a nonstandard E3-strength distribution by means of the interacting-boson model. For example, in the description of octupole states in deformed nuclei, in the IBM1+f boson model[94] the first three states with $K^{\pi}=3^-$ in ^{168}Er are simply omitted.

5.7. States with $K^{\pi}=3^+$

Low-lying states with $K^{\pi}=3^+$ have not been found in ^{156,158,160}Gd and ^{160}Dy. According to our calculations, the first 3_1^+ states in these nuclei are located in the excitation energy range 2.1–2.3 MeV. The first 3_1^+ states in ^{162}Dy, ^{164}Dy, and ^{166}Er have energies 2.283, 1.979, and 1.938 MeV. The 3_1^+ state of energy 1.653 MeV in ^{168}Er is strongly excited in the (d,d′) reaction. According to our calculations, states with $K^{\pi}=3^+$ and energy below 2.3 MeV are hexadecapole one-phonon states. Many of the wave functions of states with $K^{\pi}=3^+$ have a dominant two-quasiparticle component. In all stable nuclei with $N=98-104$ and $Z=68-72$ the first $K_n^{\pi}=3_1^+$ states are collective states.

5.8. Hexadecapole and two-phonon states with $K^{\pi}=4^+$

A state is considered to be a two-phonon state if the contribution of the two-phonon component to the wave-

TABLE XIV. Nonrotational states in ^{166}Er.

	Experiment			QPM calculations				
K^{π}_{n}	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
2^+_1	0.786	5.2		0.8	5.0		(22)1:98	
		(d,t)				$(22)_1$:	$\nu\nu 523\downarrow-521\downarrow$	24
							$\pi\pi 411\uparrow+411\downarrow$	23
							$\nu\nu 521\uparrow+521\downarrow$	16
							$\pi\pi 413\downarrow-411\downarrow$	6
							$\nu\nu 633\uparrow-651\uparrow$	5
2^-_1	1.458	5.3		1.4	5.3		$(32)_1$:97	
		(d,t):	$\nu\nu 633\uparrow-521\uparrow\approx 58\%$				$\{(20)_1,(32)_1\}$: 2	
			$\pi\pi 523\uparrow-411\uparrow\approx 4\%$			$(32)_1$:	$\nu\nu 633\uparrow-521\uparrow$	40
							$\pi\pi 523\uparrow-411\uparrow$	10
							$\nu\nu 642\uparrow-521\downarrow$	10
0^+_1	1.460	0.66		1.4	0.5		$(20)_1$:94	
							$\{(32)_1,(32)_1\}$:4	
			$\tilde{S}(t,p)=0.15$	$\tilde{S}(t,p)=0.16$		201:	$\pi\pi 411\downarrow-411\downarrow$	48
			$\tilde{S}(p,t)\leq 0.0025$	$\tilde{S}(p,t)=0.004$			$\nu\nu 512\uparrow-512\uparrow$	11
							$\nu\nu 521\downarrow-521\downarrow$	10
							$\pi\pi 523\uparrow-523\uparrow$	9
4^-_1	1.572	(^{3}He,d):	$\pi\pi 523\uparrow+411\downarrow$	1.5	1.0		$(54)_1$:97	
			large				$\{(22)_1,(32)_1\}$: 2	
		(d,t):	$\nu\nu 633\uparrow+521\downarrow\approx 4\%$			$(54)_1$:	$\pi\pi 523\uparrow+411\downarrow$	88
							$\nu\nu 633\uparrow+521\downarrow$	8
0^-_1	1.662	2.8		1.8	2.8		$(30)_1$:99	
						$(30)_1$:	$\nu\nu 642\uparrow-523\downarrow$	26
							$\nu 642\uparrow-512\uparrow$	8
							$\pi\pi 411\downarrow-541\downarrow$	2
2^+_2	(1.703)			1.9	0.3		$(22)_2$:97	
							$\{(32)_1,(54)_1\}$:2	
						$(22)_2$:	$\nu\nu 523\downarrow-521\downarrow$	60
							$\pi\pi 411\uparrow-411\downarrow$	37
0^+_2	1.713			1.8	0.4		$(20)_2$: 91; $(20)_3$: 2	
			$\tilde{S}(t,p)=0.14$	$\tilde{S}(t,p)=0.09$			$\{(22)_1,(22)_1\}$:5	
						$(20)_2$:	$\nu\nu 521\downarrow-521\downarrow$	28
			$\tilde{S}(p,t)=0.14$	$\tilde{S}(p,t)=0.04$			$\nu\nu 512\uparrow-512\uparrow$	14
							$\pi\pi 523\uparrow-523\uparrow$	14
							$\pi\pi 411\downarrow-411\downarrow$	12
1^+_1	1.812			1.8	1.9		$(21)_1$:98	
2^+1_1	1.910	(d,t):	$\nu\nu 633\uparrow-642\uparrow$			$(21)_1$:	$\nu\nu 633\uparrow-642\uparrow$	70
			large				$\pi\pi 523\uparrow-514\uparrow$	13
1^-_1	1.830		from ^{166}Ho $K^{\pi}=0^-$	1.8	3.0		311:97; 312:1	
		logft=5.2	$\nu\nu 633\uparrow-523\downarrow$			$(31)_1$:	$\nu\nu 633\uparrow-523\downarrow$	56
			large				$\nu\nu 633\uparrow-512\uparrow$	9
							$\pi\pi 523\uparrow-412\uparrow$	6
6^-_1	1.910	(d,t):	$\nu\nu 633\uparrow+523\downarrow$	1.9	0.1		$(76)_1$:100	
		(^{3}He,α):	large			$(76)_1$:	$\nu\nu 633\uparrow+523\downarrow$	97
3^-_1	1.916	(^{3}He,d):	$\pi\pi 523\uparrow-411\downarrow$	1.9	0.4		$(33)_1$:97	
			large			$(33)_1$:	$\pi\pi 523\uparrow-411\downarrow$	86
							$\nu\nu 633\uparrow-521\downarrow$	6
3^+_1	(1.938)			1.96	0.6		$(43)_1$:79; $(43)_2$:20	
						$(43)_1$:	$\nu\nu 523\downarrow+521\downarrow$	80
							$\nu\nu 512\uparrow+521\downarrow$	11
0^+_3	1.935			2.0	0.006		$(20)_3$:86; $(20)_4$:4	
			$\tilde{S}(p,t)=0.08$	$\tilde{S}(t,p)=0.014$			$(20)_2$:3; $(20)_5$:2	
							$\{(22)_1,(22)_1\}$: 4	
				$\tilde{S}(p,t)=0.0012$		$(20)_3$:	$\nu\nu 633\uparrow-633\uparrow$	40
							$\nu\nu 521\downarrow-521\downarrow$	37
4^+_1	1.979	(α,t):	$\pi\pi 523\uparrow+541\downarrow$	1.96	1.1		$(44)_1$:76	
			large				$\{(22)_1,(22)_1\}$:21	
		(d,t):	$\nu\nu 633\uparrow+660\uparrow$			$(44)_1$:	$\nu\nu 523\downarrow+521\uparrow$	37
			noticeable				$\pi\pi 523\uparrow+541\downarrow$	32
							$\nu\nu 633\uparrow+660\uparrow$	6
5^+_1				2.0	1.0		$(65)_1$:99	
						$(65)_1$:	$\nu\nu 523\downarrow+512\uparrow$	71
							$\pi\pi 404\downarrow+411\uparrow$	18
7^-_1	1.990	(^{3}He,d):	$\pi\pi 523\uparrow+404\downarrow$	2.5	-		$(77)_1$:100	

TABLE XIV. (*Continued.*)

	Experiment			QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure		%
		(α,t):	large			$(77)_1$:	$\pi\pi523\uparrow+404\downarrow$	100
2_2^-	(2.022)			2.0	0.1		$(32)_2$:99	
						$(32)_2$:	$\nu\nu642\uparrow-521\downarrow$	78
							$\pi\pi523\uparrow-411\uparrow$	12
							$\nu\nu633\uparrow-521\uparrow$	7
4_2^-	2.022	(d,t):	$\nu\nu633\uparrow+521\downarrow$	2.0	0.03		$(54)_2$:99	
			large			$(54)_2$:	$\nu\nu633\uparrow+521\downarrow$	90
							$\pi\pi523\uparrow+411\downarrow$	9
3_2^+				2.0	1.3		$(43)_2$:78; $(43)_1$:21	
						$(43)_2$:	$\nu\nu512\uparrow+521\downarrow$	47
							$\nu\nu523\downarrow+521\downarrow$	19
							$\nu\nu633\uparrow-660\uparrow$	10
							$\pi\pi404\downarrow-411\downarrow$	8
4_2^+				2.05	0.01		$\{(22)_1,(22)_1\}$:73	
							$(44)_2$: 3; $(44)_1$:23	
2_3^+				2.08	0.01		$(22)_3$:97	
							$\{(32)_1,(54)_1\}$:2	
						$(22)_3$:	$\nu\nu521\uparrow+521\downarrow$	55
							$\pi\pi411\uparrow+411\downarrow$	30
							$\nu\nu523\downarrow-521\downarrow$	8
							$\nu\nu633\uparrow-651\uparrow$	6
3_2^-	(2.080)	(d,t):	$\nu\nu\uparrow-521\downarrow$	2.1	0.1		$(33)_2$:99	
			large			$(33)_2$:	$\nu\nu633\uparrow-521\downarrow$	86
							$\pi\pi523\uparrow-411\downarrow$	9
6_1^+				2.1	0.02		$(66)_1$:100	
						$(66)_1$:	$\nu\nu633\uparrow+642\uparrow$	99.5
1_2^-				2.1	2.3		$(31)_2$:97	
						$(31)_2$:	$\nu\nu633\uparrow-523\downarrow$	44
							$\nu\nu633\uparrow-512\uparrow$	17
							$\pi\pi523\uparrow-402\uparrow$	8
3_3^+	(2.132)			2.24	0.001		$(43)_3$:95	
		$(^3He,d)$:	$\pi\pi523\uparrow-541\downarrow$				$\{(20)_1,(43)_3\}$:4	
		(α,t):	large			$(43)_3$:	$\pi\pi523\uparrow-541\downarrow$	95
		$\log ft=5.6$ from ^{166}Tm						
2_4^+	2.160		for $I^\pi K_\nu=3^+2_4$	2.2	0.002		$(22)_4$:97	
		(d,t):	$\nu\nu633\uparrow-651\uparrow$				$\{(22)_1,(44)_1\}$:1	
			large			$(22)_4$:	$\nu\nu633\uparrow-651\uparrow$	82
							$\nu\nu521\uparrow+521\downarrow$	16
0_4^+	2.196			2.1	0.01		$(20)_4$:89; $(20)_3$:3	
							$\{(22)_1,(22)_1\}$:2	
							$(20)_1$:1; $(20)_2$:1	
			$\tilde{S}(t,p)\leq0.03$		$\tilde{S}(t,p)=0.001$	$(20)_4$:	$\nu\nu523\downarrow-523\downarrow$	28
			$\tilde{S}(p,t)=0.08$		$\tilde{S}(p,t)=0.04$		$\pi\pi404\downarrow-404\downarrow$	17
							$\pi\pi523\uparrow-523\uparrow$	10
4_3^+				2.2	0.01		$(44)_2$:88	
							$\{(22)_1,(22)_1\}$:5	
						$(44)_2$:	$\pi\pi523\uparrow+541\downarrow$	60
							$\nu\nu523\downarrow+521\uparrow$	34
3_3^-				2.2	0.3		$(33)_3$:98	
							$\{(22)_1,(31)_1\}$:1	
						$(33)_3$:	$\nu\nu642\uparrow+521\downarrow$	87
0_2^-	(2.2)			2.2	0.8		$(30)_2$:99	
						$(30)_2$:	$\nu\nu642\uparrow-512\uparrow$	23
							$\nu\nu642\uparrow-523\downarrow$	21
2_3^-	(2.055)			2.2	0.3		$(32)_3$:91	
		$(^3He,d)$:	$\nu\nu523\uparrow-411\uparrow$				$\{(20)_1,(32)_1\}$:3	
		(α,t):	large				$\{(20)_2,(32)_1\}$:2	
						$(32)_3$:	$\pi\pi523\uparrow-411\uparrow$	50
							$\nu\nu633\uparrow-521\uparrow$	45
0_5^+				2.2	0.02		$(20)_5$:95	
							$(20)_4$:2 ;$(20)_3$:2	
						$(20)_5$:	$\nu\nu633\uparrow-633\uparrow$	40
							$\nu\nu512\uparrow-512\uparrow$	7
							$\nu\nu523\downarrow-523\downarrow$	6

TABLE XIV. (*Continued.*)

K^π_n	Experiment E_n MeV	Experiment $B(E\lambda)\uparrow$ one-particle units	Experiment Structure	QPM calculations E_n MeV	QPM calculations $B(E\lambda)\uparrow$ one-particle units	Structure		%
							νν404↓−404↓	4
1^+_2	(2.378)			2.2	0.01		$(21)_2$:99	
						$(21)_2$:	νν521↑−521↓	98
5^-_1	2.244			2.2	0.1		$(55)_1$:99	
		(d,t):	νν633↑+521↑			$(55)_1$:	νν633↑+521↑	89
			large				νν642↑+523↓	5
							ππ514↑+411↓	4
1^+_3				2.3	0.2		$(21)_2$:7; $(21)_3$:89	
							$\{(32)_1,(33)_1\}$:3	
						$(21)_3$:	ππ411↑−411↓	75
							ππ514↑−523↑	12
							νν521↑−52142↓	7
5^-_2				2.3	0.05		$(55)_2$:99	
						$(55)_2$:	νν523↓+642↑	89
							νν633↑+521↑	5
							ππ514↑+411↓	4
1^+_4	(2.464)			2.5	0.3		$(21)_4$:73; $(21)_5$:22	
						$(21)_4$:	ππ514↑−523↑	50
							ππ411↑−411↓	20
							νν512↑−521↑	20
6^-_2				2.4	0.02		$(76)_2$:100	
						$(76)_2$:	νν633↑+512↑	96
3^+_4	(2.293)			2.5	0.5		$(43)_4$:96	
		(d,t):	νν633↑−660↑				$\{(22)_1,(21)_1\}$:1	
			large			$(43)_4$:	νν633↑−660↑	41
							νν512↑+521↓	35
							ππ404↓−411↓	15
4^+_4	(2.318)			2.6	0.3		$(44)_3$:92	
	(2.633)						$\{(20)_1,(44)_1\}$:2	
						$(44)_3$:	νν633↑+660↑	53
							νν523↓+521↑	20
							νν642↑+651↑	11
6^-_3	(2.608)			2.6			$(76)_3$:100	
		(^{3}He,d):	ππ523↑+402↑			$(76)_3$:	ππ523↑+402↑	94
		(α,t):	large					
0^-_3	(≈2.7)			2.6	1.1		$(30)_3$:96	
							$\{(22)_1,(32)_1\}$:2	
						$(30)_3$:	νν642↑−512↑	18
							νν633↑−514↓	16
							ππ523↑−404↓	4
0^-_4				2.8	0.2		$(30)_4$:95	
						$(30)_4$:	νν633+−514↓	25
							ππ523↑−404↓	22
0^-_5	(≈2.8)			2.9	1.0		$(30)_5$:89	
							$\{(22)_1,(32)_1\}$:8	
						$(30)_5$:	ππ523↑−404↓	16
							νν633↑−514↓	7
							νν651↑−521↑	7
8^+_1	(3.075)			2.8			$(88)_1$:100	
		(^{3}He,d):	ππ523↑+514↑				ππ523↑+514↑	100
		(α,t):	large					
9^-_1	(2.494)	(d,t):	νν633↑+505↑	3.4			$(9)_1$:100	
		(^{3}He,α):	large				νν633↑+505↑	100

function normalization is more than 50%. The energy centroids of two-phonon collective states were calculated in Refs. 5 and 6, where it was shown that the inclusion of the Pauli principle in the two-phonon components of the wave functions leads to a shift of the two-phonon poles to higher excitation energies, where the level density is large. Therefore, two-phonon collective states must be strongly fragmented. In Refs. 5 and 6 it was stated that two-phonon states consisting of two collective phonons should not exist in strongly deformed nuclei of the rare-earth region. In most cases this prediction is correct. In our earlier calculations (Refs. 5, 6, 78, and 93) the shift of the two-phonon poles was too large, especially for states $K^\pi=4^+\{(22)_1,(22)_1\}$, where a factor of 1/2 was omitted. In the more accurate calculations

TABLE XV. E1 and M1 transitions to the ground state and E1, M1, and E2 transitions between excited states in ^{166}Er.

Initial state		Eλ or M1	Final state			$B(E\lambda)\downarrow$, e^2 $F^{2\lambda}$ or $B(M1)\downarrow$, μ_N^2	
$I^\pi K_n$	E_n, MeV		n_f	$I^\pi K_n$	E_n, MeV	exp. (Ref. 49)	calculation
0^+0_1	1.460	E2		2^+2_1	0.786		1.1
1^-0_1	1.662	E1		$0^+0_{g.s.}$	0	$(8.9\pm0.5)\cdot10^{-3}$	$30\cdot10^{-3}$
0^+0_2	1.713	E2		2^+2_1	0.786		95
1^+1_1	1.812	M1		$0^+0_{g.s.}$	0		0.9
1^-1_1	1.830	E1		$0^+0_{g.s.}$	0	$\approx1\cdot10^{-3}$	$7\cdot10^{-3}$
3^-3_1	1.918	E1	1	2^+2_1	0.786		$7\cdot10^{-6}$
		M1	2	2^-2_1	1.458		0.03
0^+0_3	1.935	E2		2^+2_1	0.786		60
4^+4_1	1.978	E2		2^+2_1	0.786		115
4^+4_2	2.05*	E2		2^+2_1	0.786		500
3^+3_2	2.133	M1	1	2^+2_1	0.786		0.001
		E2					0.75
		E1	2	2^-2_1	1.458		10^{-4}
		E1	3	4^-4_1	1.572		$3\cdot10^{-6}$
		M1	4	2^+2_1	1.703		$4\cdot10^{-4}$
		E1	5	3^-3_1	1.916		$2\cdot10^{-5}$
		E1	6	4^-4_2	2.002		$3\cdot10^{-6}$
3^+2_4	2.160	E2	1	$2^+0_{g.s.}$	0.081		0.08
		M1	2	2^+2_1	0.786		$9\cdot10^{-4}$
		E2					0.05
		E2	3	2^+0_1	1.528		0.11
1^-0_2	≈2.2	E1		$0^+0_{g.s.}$	0	$\approx3\cdot10^{-3}$	$7\cdot10^{-3}$
4^+4_3	2.2*	E2		2^+2_1	0.786		70
3^-3_2	2.216	E1	1	2^+2_1	0.786		10^{-4}
		M1	2	2^-2_1	1.458		0.01
		M1	3	4^-4_1	1.572		$5\cdot10^{-5}$
		E2	4	1^-1_1	1.830		0.12
		M1	5	3^-3_1	1.916		$6\cdot10^{-3}$
		M1	6	4^-4_2	2.002		$3\cdot10^{-4}$
3^-3_3	2.243	E1	1	2^+2_1	0.786		10^{-5}
		M1	2	3^-2_1	1.514		$2\cdot10^{-4}$
4^+4_4	2.6*	E2		2^+2_1	0.786		0.42
1^-0_3	≈2.7	E1		$0^+0_{g.s.}$	0	$1\cdot10^{-3}$	$12\cdot10^{-3}$
1^-0_4	≈2.8	E1		$0^+0_{g.s.}$	0	$4\cdot10^{-3}$	$8\cdot10^{-3}$

*Calculated energies.

where the pp interaction is included along with the ph one, the shift of the two-phonon poles consisting of two collective phonons was 0.5–1.0 MeV.

In Refs. 18, 23, and 79 it was stated that the nuclei ^{164}Dy and ^{166,168}Er are the most favorable for discovering doubly gamma-vibrational states with $K^\pi=4^+$ in the energy range 2.0–2.3 MeV. The experimental studies of Refs. 72, 73, and 95 showed that there is a large two-phonon doubly gamma-vibrational component in the first $K_n^\pi=4_1^+$ state in ^{168}Er. According to the calculations of Ref. 23, in the QPM the contributions of the hexadecapole one-phonon component $(44)_1$ and two-phonon component $\{(22)_1,(22)_1\}$ to the normalization of the state with $K_n^\pi=4_1^+$ in ^{168}Er are 60% and 30%, respectively. The calculated energies of the states with $K_n^\pi=2_1^+$, $K_n^\pi=4_1^-$, and $K_n^\pi=4_1^+$ and also the values of $B(E2;2^+2_1\to0^+0_{g.s.})$, $B(E4;4^+4_1\to0^+0_{g.s.})$, $B(M2;4^-4_1\to2^+2_1)$, and $B(E1;4^+4_1\to4^-4_1)$ are in good agreement with the experimental data. This is shown in Tables XVI and XVII. The calculated ratio

$$\frac{B(E2;2^+2_1\to4^+4_1)}{B(E2;0^+0_{g.s.}\to2^+2_1)},$$

equal to 0.26, is consistent with the experimental values 0.40 ± 0.20 (Ref. 72) and 0.53 ± 0.12 (Ref. 95).

Let us consider the situation regarding two-phonon states in ^{166}Er. In the spectrum of nonrotational states in ^{166}Er there is a gap between the first state with $K_n^\pi=2_1^+$ and the next state with $K_n^\pi=2_2^+$ equal to 0.672 MeV. Because of this gap, the density of two-phonon poles up to excitation energies of 4 MeV is small, and up to 3 MeV there are only five poles. Therefore, the contribution of two-phonon configurations to the normalization of the wave functions of states with $K^\pi\neq4^+$ and 0^+ and energies below 2.3 MeV is less than 6%. Owing to the small density of levels with $K_n^\pi=4^+$ near the pole $\{(22)_1,(22)_1\}$ and the small numerical value of the function $U_{221,221}^{441}$ connecting the one- and two-phonon configurations, the two-phonon state $4^+\{(22)_1,(22)_1\}$ is weakly fragmented. On the basis of this,

TABLE XVI. Nonrotational states in ^{168}Er.

	Experiment				QPM calculations				
K^π_n	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	%	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
2^+_1	0.821	4.7	$(\vec{t},\alpha)$:ππ413↓−411↓	50	0.8	4.4		$(22)_1$:94	
			ππ411↑+411↓	37				$\{(22)_1,(20)_1\}$:1	
			logft=5.2:				$(22)_1$:	ππ411↑+411↓	30
			νν523↓−521↓					ππ413↓−411↓	26
			noticeable					νν521↑+521↓	23
								νν523↓−521↓	16
4^-_1	1.094		(d,p):νν633↑+521↓	70	1.0	2.5		$(54)_1$:99	
			$(\vec{t},\alpha)$:ππ411↓+523↑	25			$(54)_1$:	νν633↑+521↓	66
								ππ523↑+411↓	20
0^+_1	1.217	≤0.1	(t,d):νν633↑−633↑	60	1.3	0.07		$(20)_1$:64;$(20)_2$:23	
								$\{(22)_1,(22)_1\}$:4	
								$\{(32)_1,(32)_1\}$:2	
							$(20)_1$:	νν512↑−512↑	36
								νν633↑−633↑	34
								νν521↓−521↓	14
								ππ411↓−411↓	2
1^-_1	1.358	3.92	(d,t):νν512↑−633↓	80	1.3	4.3		$(31)_1$:94	
			(d,p):νν512↑−633↓	80				$\{(22)_1,(33)_2\}$:3	
							$(31)_1$:	νν512↑−633↑	76
0^+_2	1.422		(t,d):νν633↑−633↑	≤20	1.4	0.03		$(20)_2$:64;$(20)_1$:27	
								$\{(22)_1,(22)_1\}$:3	
							$(20)_2$:	νν521↓−521↓	55
								νν633↑−633↑	22
								νν512↓−512↓	21
3^-_1	1.542	0.25	(d,p):νν633↑−521↓	90	1.6	0.3		$(33)_1$:98	
			$(\vec{t},\alpha)$:ππ523↑−411↓	4			$(33)_1$:	νν633↑−521↓	92
								ππ523↑−411↓	2
2^-_1	1.569	4.94			1.6	4.7		$(32)_1$:94	
								$\{(20)_1,(32)_1\}$:2	
							$(32)_1$:	νν633↑−521↑	29
								ππ523↑−411↑	20
								νν624↑−512↑	11
3^+_1	1.653		(d,d') large for 4^+3_1		1.6	0.8		$(43)_1$:98	
							$(43)_1$:	νν512↑+521↓	91
0^-_1	1.786	1.96			1.8	3.0		$(30)_1$:98	
							$(30)_1$:	νν642↑−512↑	25
								νν514↓−633↑	7
								ππ523↑−404↓	3
3^-_2	1.828	0.60			1.9	0.5		$(33)_2$:92	
							$(33)_2$:	νν633↑−510↑	80
								ππ523↑−411↓	6
								ππ514↑−411↑	3
0^+_3	1.833		$(\vec{t},\alpha)$:ππ411↓−411↓	25	1.8	0.02		$(20)_3$:94	
								$\{(32)_1,(32)_1\}$:3	
							$(20)_3$:	ππ411↓−411↓	44
								ππ523↑−523↑	28
								ππ411↑−411↑	9
								νν512↑−512↑	3
								νν633↑−633↑	2
2^+_2	1.848				1.8	0.02		$(22)_2$:96	
							$(22)_2$:	νν512↑−521↓	97
4^-_2	1.905		$(\vec{t},\alpha)$:ππ411↓+523↑	60	1.8	0.9		$(54)_2$:99	
			(d,p):νν633↑+521↓	30			$(54)_2$:	ππ411↓+523↑	60
								νν633↑+521↓	31
2^+_3	1.930		logft=6.2:		2.0	0.2		$(22)_3$:68; $(22)_5$:8	
			νν523↓−521↓					$\{(22)_1,(20)_1\}$:9	
			small					$\{(22)_1,(44)_1\}$:3	
							$(22)_3$:	νν521↑+521↓	63
								ππ411↑+411↓	16
1^-_2	1.937		$(\vec{t},\alpha)$ small		1.9	0.4		$(31)_2$:96	
							$(31)_2$:	νν633↑−523↓	85
								νν633↑−512↑	9
3^-_3	1.999	0.42	$(\vec{t},\alpha)$:ππ523↑−411↓	75	2.0	0.6		$(33)_3$:96	
			(d,p):νν633↑−510↑	10			333:	ππ523↑−411↓	76

TABLE XVI. (*Continued.*)

	Experiment				QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	%	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
								$\nu\nu633\uparrow-510\uparrow$	13
								$\pi\pi514\uparrow-411\uparrow$	6
4_1^+	2.056	0.6			2.0	0.9		$(44)_1$:60	
								$\{(22)_1,(22)_1\}$:30	
								$\{(20)_1,(44)_1\}$:2	
							$(44)_1$:	$\pi\pi523\uparrow+541\downarrow$	50
								$\nu\nu512\uparrow+512\downarrow$	9
								$\nu\nu514\downarrow+521\downarrow$	8
4_3^-	2.060		(d,p):$\nu\nu633\uparrow+510\uparrow$	33	2.3	0.002		$(54)_3$:92	
			(t,d):$\nu\nu633\uparrow+510\uparrow$	33				$\{(22)_2,(32)_1\}$:5	
							$(54)_3$:	$\nu\nu624\uparrow-521\downarrow$	98
7_1^-	2.122				2.2	0.02		$(77)_1$:100	
							$(77)_1$:	$\pi\pi523\uparrow+404\downarrow$	95
								$\nu\nu514\downarrow+633\uparrow$	4
1_1^+	2.134				2.1	0.2		$(21)_1$:82; $(21)_3$:7	
								$\{(31)_1,(32)_1\}$:5	
								$\{(33)_2,(54)_1\}$:3	
							$(21)_1$:	$\nu\nu633\uparrow-642\uparrow$	80
								$\nu\nu6243\uparrow-633\uparrow$	13
								$\pi\pi514\uparrow-523\uparrow$	3
3_2^+	2.187				2.3	0.007		$(43)_2$:99	
							$(43)_2$:	$\nu\nu523\downarrow+521\downarrow$	91
2_4^+	2.193		$(\vec{t},\alpha)$:$\pi\pi411\uparrow+411\downarrow$	20–30	2.1	0.1		$(22)_4$:52	
			$\log ft=4.8$:					$(22)_3$:18; $(22)_5$:11	
			$\nu\nu523\downarrow-521\downarrow$					$\{(22)_1,(20)_1\}$:12	
			large					$\{(22)_1,(44)_1\}$:6	
							$(22)_4$:	$\nu\nu523\downarrow-521\downarrow$	60
								$\pi\pi411\uparrow+411\downarrow$	29
2_2^-	2.230				2.2	0.07		$(32)_2$:94	
								$\{(20)_1,(32)_1\}$:2	
							$(32)_2$:	$\nu\nu642\uparrow-521\downarrow$	83
4_2^+	2.238				2.4	0.2		$(44)_2$:47; $(44)_1$:23	
								$\{(22)_1,(22)_1\}$:31	
							$(44)_2$:	$\pi\pi523\uparrow+541\downarrow$	35
								$\nu\nu514\downarrow+521\downarrow$	33
3_4^-	2.263	4.68			2.3	2.4		$(33)_4$:65	
								$(33)_8$:3; $(33)_9$:4	
								$\{(22)_1,(31)_1\}$:21	
							$(33)_4$:	$\nu\nu642\uparrow+521\downarrow$	39
								$\pi\pi514\uparrow-411\uparrow$	23
								$\pi\pi523\uparrow-411\downarrow$	16
5_1^+	2.268				2.0	1.2		$(65)_1$:100	
							$(65)_1$:	$\pi\pi404\downarrow+411\uparrow$	55
								$\nu\nu523\downarrow+512\uparrow$	29
								$\nu\nu514\downarrow+521\uparrow$	4
5_2^+	(2.298)				2.3	0.04		$(65)_2$:100	
							$(65)_2$:	$\nu\nu523\downarrow+512\uparrow$	68
								$\pi\pi404\downarrow+411\uparrow$	30
3_5^-	2.323	1.53			2.4	1.1		$(33)_4$:25; $(33)_8$:7	
								$(33)_9$:7	
								$\{(22)_1,(31)_1\}$:60	
1_2^+	2.365		$(\vec{t},\alpha)$:$\pi\pi411\uparrow+411\downarrow$		2.3	$4\cdot10^{-4}$		$(21)_2$:93	
			large					$\{(32)_1,(33)_3\}$:2	
							$(21)_2$:	$\pi\pi411\uparrow-411\downarrow$	93
5_1^-	2.366				2.2	0.3		$(55)_1$:100	
							$(55)_1$:	$\nu\nu521\uparrow+624\uparrow$	60
								$\pi\pi514\uparrow+411\downarrow$	31
								$\pi\pi523\uparrow+411\uparrow$	5
2_5^+	2.425		$(\vec{t},\alpha)$:$\pi\pi411\uparrow+411\downarrow$		2.4	0.01		$(22)_5$:51	
			noticeable					$(22)_4$:30; $(22)_6$:12	
			$\log ft=4.6$:				$(22)_5$:	$\nu\nu633\uparrow-651\uparrow$	80
			$\nu\nu523\downarrow-521\downarrow$					$\pi\pi411\uparrow+411\downarrow$	8
			large					$\nu\nu523\downarrow-521\downarrow$	5

TABLE XVI. (*Continued.*)

	Experiment				QPM calculations				
K_n^π	E_n MeV	$B(E\lambda)\uparrow$ one-particle units	Structure	%	E_n MeV	$B(E\lambda)\uparrow$ one-particle units		Structure	%
4_3^+	2.663				2.6	0.2		$(44)_3$:42; $(44)_4$:20	
								$(44)_2$:18	
								$\{(22)_1,(22)_1\}$:10	
								$\{(20)_1,(44)_1\}$:2	
							$(44)_3$:	$\nu\nu514\downarrow+521\downarrow$	47
								$\nu\nu512\uparrow+521\uparrow$	33
								$\nu\nu523\uparrow+541\downarrow$	6

the existence of a doubly gamma-vibrational state with $K_n^\pi=4_1^+$ and energy 2.05 MeV was predicted in Ref. 18.

Searches for a doubly gamma-vibrational state with $K^\pi=4^+$ in ^{166}Er were carried out in Refs. 55 and 96 in Coulomb-excitation experiments using a ^{58}Ni beam. The authors of Ref. 96 found a small part of the two-phonon configuration $\{(22)_1,(22)_1\}$ in the first $K_n^\pi=4_1^+$ state with energy 1.978 MeV and a large part in the second $K_n^\pi=4_2^+$ state with energy 2.029 MeV. These experimental data confirm the prediction of a two-phonon state in ^{166}Er made in Ref. 18 on the basis of the QPM calculations.

The situation regarding the doubly gamma-vibrational state with $K^\pi=4^+$ in ^{164}Dy remains unclear. According to our calculations, a large part of the strength of the $4^+\{(22)_1,(22)_1\}$ state is concentrated on one or two states with $K^\pi=4^+$ in the energy range 2.1–2.3 MeV. In Ref. 55 it was shown that the 4^+ state of energy 2.206 MeV in ^{164}Dy is apparently a two-phonon state. A doubly gamma-vibrational state with $K^\pi=4^+$ in ^{164}Dy was sought in Ref. 97. From the results of the measurements it was concluded that there is no doubly gamma-vibrational state with excitation energy below 2.06 MeV. The study of Ref. 97 did not exclude the possibility that the collective two-phonon state has energy 2.206 MeV.

The first states with $K_n^\pi=4_1^+$ in ^{156}Gd and a number of other nuclei were interpreted in Refs. 98 and 99 as doubly gamma-vibrational states. This interpretation is based on E2 transitions to the gamma-vibrational state. According to our calculations, the first and second states with $K^\pi=4^+$ in ^{156,158,160}Gd and in ^{160,162}Dy are hexadecapole states. Large admixtures of two-phonon components $\{(22)_1,(22)_1\}$ are responsible for the fairly fast E2 transitions from $K_n^\pi=4_1^+$ to $K_n^\pi=2_1^+$ states. Large two-quasiparticle components $\nu\nu523\downarrow+521\uparrow$ and $\pi\pi413\downarrow+411\uparrow$ in the one-phonon terms of their wave functions were found in (^{3}He,α), (α, ^{3}He), (t,α), and (d,p) reactions and in β decays. The results of the calculations and their comparison with the experimental data (Tables II–VI and VIII–XI) show that the first and second $K^\pi=4^+$ states cannot be interpreted as two-phonon states. As was shown in Ref. 100, all the available experimentaldata, from E4 transitions to ground states, one-nucleon transfer reactions, and β decays, suggest that states with $K^\pi=4^+$ in these nuclei are mainly hexadecapole vibrational states.

5.9. States with λ>5

There is a considerable amount of experimental data on states with $K^\pi=4^-$. For example, in ^{168}Er there are three one-phonon states with $K^\pi=4^-$, where the first two are excited in (d,p) and ($\vec{t}$,α) reactions. The first 4_1^- state has a large two-quasineutron component, and the second 4_2^- state has a large two-quasiproton component. Almost all the first two states with $K^\pi=4^-$ are one-phonon states with two large two-quasiparticle components. The QPM calculations give a fairly good description of the energies and structure of states with $K^\pi=4^-$.

Experimental data are available on states with $K^\pi=5^+$, 5^-, 6^-, and 7^-. Most of these are two-quasiparticle states. In some of them there are several small components in addition to a large two-quasiparticle one. As was demonstrated in Ref. 80, when calculating states of high multipole order it is necessary to take into account the corresponding multipole–multipole interactions.

6. CONCLUSION

On the basis of the QPM calculations of nonrotational states in even–even deformed nuclei, the results of which are summarized in this review, and comparison of the results with the corresponding experimental data, we arrive at the following conclusions.

1. The QPM gives a fairly good description of the available experimental data on the energies and structure of nonrotational states of ^{156,158,160}Gd, ^{160,162,164}Dy, and ^{166,168}Er, and predictions have been made using this model. All the nonrotational states with excitation energy below 2.3 MeV have been calculated.

2. It is practically impossible to separate collective vibrational states, except for gamma-vibrational ones, from less collective and two-quasiparticle states. Phenomenological models are based on this separation. In contrast, the QPM uses a unified basis for describing all nonrotational states.

3. The wave functions of all excited states with energy below 2.3 MeV, except for states with $K^\pi=4^+$ in ^{164}Dy and ^{166,168}Er, have a dominant one-phonon term. The contribution of two-phonon configurations to the normalization of their wave functions is less than 10%.

4. The sizable cross sections for one-nucleon transfer reactions may be due to the corresponding large two-

TABLE XVII. E1 transitions to the ground state and E1, E2, M1, and M2 transitions between excited states in ^{168}Er.

Initial state		Eλ or Mλ	Final state			$B(E\lambda)\downarrow$, e^2 F$^{2\lambda}$ or $B(M\lambda)\downarrow$, μ_N^2 F$^{2\lambda-2}$	
$I^\pi K_n$	E_n, MeV		n_f	$I^\pi K_n$	E_n, MeV	exp. [Ref.]	calculation
4^-4_1	1.094	M2	1	2^+2_1	0.821	0.42 71	0.6
2^+0_1	1.276	E2	1	$0^+0_{g.s.}$	0.	≤4 69	4
		E2	2	$2^+0_{g.s.}$	0.079		5.7
		E2	3	2^+2_1	0.821		32
1^-1_1	1.358	E1	1	$0^+0_{g.s.}$	0.	$1.5\cdot10^{-3}$ 71	$5\cdot10^{-3}$
3^-1_1	1.431	E1	1	$2^+0_{g.s.}$	0.079	$2.2\cdot10^{-6}$ 71	$7\cdot10^{-3}$
		E1	2	3^+2_1	0.895	$1.2\cdot10^{-7}$ 71	$5\cdot10^{-5}$
2^+0_2	1.496	E2	1	$2^+0_{g.s.}$	0.079		2.0
		E2	2	2^+2_1	0.821		18
		E2	3	2^+0_1	1.276		$4\cdot10^{-4}$
3^-3_1	1.542	E1	1	2^+2_1	0.821	$4.1\cdot10^{-5}$ 71	$6\cdot10^{-5}$
		M1	2	4^-4_1	1.094	$3.0\cdot10^{-2}$ 71	10^{-31}
2^-2_1	1.569	E1	1	2^+2_1	0.821		$4\cdot10^{-3}$
3^+3_1	1.653	M1	1	2^+2_1	0.821		$8\cdot10^{-4}$
		E1	2	4^-4_1	1.094		$6\cdot10^{-6}$
		E1	3	3^-3_1	1.542		10^{-6}
		E1	4	2^-2_1	1.569		10^{-5}
1^-0_1	1.786	E1	1	$0^+0_{g.s.}$	0	$9\cdot10^{-3}$ 49	$3\cdot10^{-2}$
3^-3_2	1.828	E1	1	2^+2_1	0.821		$3\cdot10^{-6}$
		E2	2	1^-1_1	1.358		8
		M1	3	3^-3_1	1.542		0.002
		M1	4	4^-3_1	1.615		$7\cdot10^{-4}$
2^+2_2	1.848	E2	1	$0^+0_{g.s.}$	0		1.5
		M1	2	2^+2_1	0.821		0.004
		E2	3	2^+0_1	1.276		0.06
		E2	4	2^+0_2	1.493		1.2
		E1	5	2^-2_1	1.569		$2\cdot10^{-4}$
		M1	6	3^+3_1	1.653		$8\cdot10^{-6}$
2^+0_3	1.893	E2	1	$2^+0_{g.s.}$	0.079		3.2
		E2	2	3^+2_1	0.895		9
4^-4_2	1.905	M1	1	4^-4_1	1.094		0.05
		M1	2	3^-3_1	1.542		10^{-4}
2^+2_3	1.930	E2	1	$0^+0_{g.s.}$	0		14
		M1	2	2^+2_1	0.821		$3\cdot10^{-3}$
		E2					0.1
		M1	3	3^+3_1	1.653		$8\cdot10^{-4}$
1^-1_2	1.937	E1	1	$0^+0_{g.s.}$	0	$2.2\cdot10^{-4}$ 71	$6\cdot10^{-4}$
		M1	2	1^-1_1	1.358		$6\cdot10^{-5}$
		M1	2	1^-0_1	1.786		10^{-3}
2^-1_2	1.972	E1	1	2^+2_1	0.821		$8\cdot10^{-7}$
		E2	2	3^-3_1	1.542		4.7
3^-3_3	1.999	M1	1	3^-3_1	1.542	$5.8\cdot10^{-4}$ 71	$8\cdot10^{-3}$
		M1	2	2^-2_1	1.569	$2.5\cdot10^{-4}$ 71	0.02
		E1	3	3^+3_1	1.653		$4\cdot10^{-7}$
		M1	4	3^-3_2	1.828		0.01
4^+4_1	2.056	E2	1	2^+2_1	0.821	280±140 72	175
						315 73	
		E1	2	4^-4_1	1.094	$5.5\cdot10^{-4}$ 74	$8\cdot10^{-4}$
4^-4_3	2.060	M1	1	4^-4_1	1.094		$2\cdot10^{-3}$
		M1	2	5^-3_1	1.707		$5\cdot10^{-3}$
		M1	3	4^-3_2	1.892		$2\cdot10^{-3}$
		M1	4	4^-4_2	1.905		$6\cdot10^{-3}$
		E2					0.05
3^+3_2	2.187	E1	1	3^-3_1	1.542		$6\cdot10^{-5}$
		M1	2	3^+3_1	1.653		$5\cdot10^{-3}$
2^+2_4	2.193	M1	1	2^+2_1	0.821		$7\cdot10^{-8}$
		E2					$2\cdot10^{-3}$
2^-2_2	2.230	E1	1	2^+2_1	0.821		$4\cdot10^{-4}$
		M1	2	3^-3_1	1.542		$8\cdot10^{-3}$
		M1	3	2^-2_1	1.569		$5\cdot10^{-3}$
4^+4_2	2.238	E1	1	4^-4_1	1.094		10^{-3}

TABLE XVII. (*Continued.*)

Initial state		Eλ or Mλ	Final state			$B(E\lambda)\downarrow$, e^2 F$^{2\lambda}$ or $B(M\lambda)\downarrow$, μ_N^2 F$^{2\lambda-2}$	
$I^\pi K_n$	E_n, MeV		n_f	$I^\pi K_n$	E_n, MeV	exp. [Ref.]	calculation
		M1	2	4^+3_1	1.736		10^{-3}
		E2	3	2^+2_2	1.848		0.4
		E1	4	4^-3_2	1.892		10^{-5}
		E1	5	4^-4_2	1.905		10^{-3}
3^-3_4	2.263	E1	1	2^+2_1	0.821		$4\cdot10^{-6}$
		M1	2	4^-3_1	1.615		$5\cdot10^{-4}$
		E2					0.11
		M1	3	3^-2_1	1.633		$4\cdot10^{-3}$
		E1	4	3^+3_1	1.653		$3\cdot10^{-5}$
		M1	5	3^-3_3	1.999		10^{-3}
2^+2_5	2.425	E2	1	$0^+0_{g.s.}$	0		5.6
		M1	2	2^+2_1	0.821		$8\cdot10^{-3}$
		E2	3	0^+0_1	1.217		31.5
4^+4_3	2.663	E1	1	4^-4_1	1.094		$3\cdot10^{-5}$
		M1	2	3^+3_1	1.653		0.01

quasiparticle configurations of one-phonon terms of the excited-state wave functions.

5. The first excited 0_1^+ states in ^{162,164}Dy and ^{166,168}Er cannot be treated as beta-vibrational states, owing to the very small probabilities of E2 transitions to the ground-state rotational band. In these nuclei the reduced probabilities of E2 transitions to the gamma-vibrational band dominate over E2 transitions to the ground state. This dominance arises from the small reduced probability for the E2 transition to the ground-state band and the 2–4% admixture of the doubly gamma-vibrational configuration in the wave function of the 0_1^+ state.

6. The nuclei ^{164}Dy and ^{166,168}Er are the most favorable of the even–even nuclei in the rare-earth region for observing doubly gamma-vibrational states with $K^\pi=4^+$ in the energy range 2.0–2.3 MeV.

7. The wave functions of the first and second states with $K^\pi=4^+$ in ^{156,158,160}Gd and ^{160,162}Dy have a dominant one-phonon hexadecapole term.

8. The reduced probabilities $B(\mathrm{E1};0^+0_{g.s.}\to 1^-K_n)$ for transitions to states with $K^\pi=0^-$ and 1^- are mainly determined by the isoscalar octupole–octupole and isovector dipole–dipole ph interactions. The inclusion of the dipole-dipole interaction leads to a shift of a large part of the E1 strength from low-lying states to the region of the isovector giant dipole resonance.

9. The calculated reduced probabilities $B(\mathrm{E1};0^+0_{g.s.}\to 1^-K_n)$ with $K^\pi=0^-$ and 1^- are 3–5 times larger than the experimental values. The full E1 strength up to 3 MeV for excited states with $K^\pi=0^-$ is 3–4 times larger than for the excitation of states with $K^\pi=1^-$. There are strong correlations between the values of B(E1) and B(E3) for γ transitions to a given rotational band.

10. According to our calculations, there should be fast E1 and M1 transitions between the large components of the wave functions of excited states differing by an octupole phonon with $K^\pi=0^-$ or 1^- or a quadrupole phonon with $K^\pi=1^+$.

11. The reduced probabilities for Eλ and Mλ transitions between one-phonon terms of the wave functions depend strongly on the small components, so that in some cases their description cannot be considered satisfactory. The intensities of M1 transitions are larger than those of the corresponding E2 transitions.

12. The Coriolis interaction is taken into account only when necessary. Our wave functions can be used to calculate the intensities of γ transitions between rotational bands, taking into account the Coriolis interaction.

13. The fragmentation and mixing of one-phonon states is enhanced as the excitation energy increases. This must be taken into account when describing levels with energies above 2.0–2.5 MeV.

14. To understand the properties of deformed nuclei it is necessary to carry out experimental studies of nonrotational states in the energy range 2–4 MeV.

This work was carried out with the support of the RFFI (grant 94–02–05137a).

[1] C. J. Gallagher and V. G. Soloviev, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **2**, No. 2 (1962).
[2] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975) [Russ. transl., Mir, Moscow, 1985].
[3] V. G. Solov'ev, *Theory of Complex Nuclei* (Pergamon Press, Oxford, 1976) [Russ. original, Nauka, Moscow, 1971].
[4] E. P. Grigor'ev and V. G. Solov'ev, *Structure of Even Deformed Nuclei* [in Russian] (Nauka, Moscow, 1974).
[5] V. G. Solov'ev and N. Yu. Shirikova, Z. Phys. A **301**, 263 (1981).
[6] V. G. Solov'ev and N. Yu. Shirikova, Yad. Fiz. **36**, 1376 (1982) [Sov. J. Nucl. Phys. **36**, 799 (1982)].
[7] D. Bohle, A. Richter, W. Steffen *et al.*, Phys. Lett. **137B**, 27 (1984).
[8] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
[9] I. Hamamoto and C. Magnusson, Phys. Lett. **312B**, 267 (1992).
[10] R. Nojarov and A. Fassler, Nucl. Phys. **A484**, 1 (1988).
[11] A. Fassler, R. Nojarov, and T. Taigel, Nucl. Phys. **A492**, 105 (1989).
[12] D. Zawischa and J. Speth, Z. Phys. A **542**, 375 (1992).
[13] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Fiz. Élem. Chastits

At. Yadra **25**, 377 (1994) [Phys. Part. Nucl. **25**, 157 (1994)].
[14] V. G. Solov'ev, Fiz. Élem. Chastits At. Yadra **9**, 860 (1978) [Sov. J. Part. Nucl. **9**, 343 (1978)].
[15] V. G. Solov'ev, *Theory of the Nucleus. Quasiparticles and Phonons* (Institute of Physics, Bristol, England, 1992) [Russ. original, Énergoatomizdat, Moscow, 1989].
[16] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Nucl. Phys. **A568**, 244 (1994).
[17] D. Karadjov, V. V. Voronov, and F. Catara, J. Phys. G **20**, 1431 (1994).
[18] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Phys. Rev. C **51**, 551 (1995).
[19] J. Berzins, P. Prokofjevs, R. Georgii *et al.*, Nucl. Phys. **A548**, 413 (1995).
[20] M. I. Baznat and N. I. Pyatov, Yad. Fiz. **21**, 708 (1975) [Sov. J. Nucl. Phys. **21**, 365 (1975)].
[21] V. M. Strutinskiĭ, Yad. Fiz. **3**, 614 (1966) [Sov. J. Nucl. Phys. **3**, 449 (1966)].
[22] C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
[23] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, J. Phys. G **20**, 113 (1994).
[24] V. G. Solov'ev and A. V. Sushkov, Z. Phys. A **345**, 155 (1993).
[25] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Int. J. Mod. Phys. E **3**, 1227 (1994).
[26] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Yad. Fiz. **59**, 57 (1996) [Phys. At. Nucl. **59**, 51 (1996)].
[27] R. C. Greenwood, C. W. Reich, H. A. Baader *et al.*, Nucl. Phys. **A304**, 327 (1978).
[28] A. Bäckllin, G. Hedin, B. Fogelberg *et al.*, Nucl. Phys. **A380**, 189 (1982).
[29] J. Klora, H. G. Börner, T. von Egidy *et al.*, Nucl. Phys. **A561**, 1 (1993).
[30] M. K. McGowan and W. T. Milner, Phys. Rev. C **23**, 1926 (1981).
[31] D. G. Burke, E. Hammaren, C. L. Swift *et al.*, Nucl. Phys. **A366**, 202 (1981).
[32] J. Konijn, F. W. N. de Boer, A. van Poelgeest *et al.*, Nucl. Phys. **A352**, 191 (1981).
[33] D. Bohle, A. Richter, K. Heyde *et al.*, Phys. Rev. Lett. **55**, 1661 (1985).
[34] G. Lovhoiden, T. F. Thorsteinsen, A. Andersen *et al.*, Nucl. Phys. **A494**, 157 (1989).
[35] H. H. Pitz, U. E. P. Berg, R. D. Heil *et al.*, Nucl. Phys. **A492**, 411 (1989).
[36] H. Friedrichs, D. Hager, P. von Brentano *et al.*, Nucl. Phys. **A567**, 266 (1994).
[37] C. W. Reich, Nucl. Data Sheets **68**, 405 (1993).
[38] G. Lovhoiden, T. F. Thorsteinsen, and D. G. Burke, Phys. Scr. **34**, 691 (1986).
[39] M. Sugaware, H. Kusakari, T. Morikawa *et al.*, Nucl. Phys. **A557**, 653c (1993).
[40] A. A. Aleksandrov, V. S. Buttsev, Ts. Vylov *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **38**, 77 (1974) [Bull. Acad. Sci. USSR, Phys. Ser.].
[41] R. E. McAdam and O. H. Otterson, Z. Phys. **250**, 2096 (1972).
[42] T. Grotdal, K. Nybo, T. Thorsteinsen, and B. Elbek, Nucl. Phys. **A110**, 385 (1968).
[43] J. Gen-ming, J. D. Garrett, G. Lovhoiden *et al.*, Phys. Rev. Lett. **46**, 222 (1981).
[44] T. Ramsoy, J. Rekstad, M. Guttorsen *et al.*, Nucl. Phys. **A438**, 301 (1985).
[45] D. G. Burke, G. Lovhoiden, and T. Thorsteinsen, Nucl. Phys. **A483**, 221 (1988).
[46] H. J. Riezebos, M. J. A. De Voigt, C. A. Fields *et al.*, Nucl. Phys. **A465**, 1 (1987).
[47] C. Wesselborg, P. von Brentano, K. O. Zell *et al.*, Phys. Lett. **207B**, 22 (1988).
[48] A. J. Cresswell, P. A. Butler, D. Cline *et al.*, Phys. Rev. C **52**, 1934 (1995).
[49] A. Zilges, P. von Brentano, H. Friedrichs *et al.*, Z. Phys. A **340**, 155 (1991).
[50] R. G. Helmer, Nucl. Data Sheets **64**, 79 (1994).
[51] D. D. Warner, R. F. Casten, W. R. Kane, and W. Gelletly, Phys. Rev. C **27**, 2292 (1989).
[52] H. Friedrichs, B. Schlitt, J. Margraf *et al.*, Phys. Rev. C **45**, R892 (1992).
[53] J. Kern, O. Mikoshiba, R. K. Sheline *et al.*, Nucl. Phys. **A104**, 642 (1967).
[54] S. J. Freeman, R. Chapman, J. L. Durell *et al.*, Nucl. Phys. **A554**, 333 (1993).
[55] M. Oshima, T. Morikawa, Y. Hatsukawa *et al.*, in *Proc. of the Fourth Intern. Conf. on Selected Topics in Nuclear Structure*, edited by V. G. Soloviev, Report E4-94-370, JINR, Dubna (1995), p. 154.
[56] E. N. Shurshikow, Nucl. Data Sheets **47**, 433 (1986).
[57] E. N. Shurshikow and N. V. Timofeeva, Nucl. Data Sheets **65**, 365 (1992).
[58] F. Hoyler, private communication.
[59] E. N. Shurshikow and N. V. Timofeeva, Nucl. Data Sheets **67**, 45 (1992).
[60] E. P. Grigor'ev, Yad. Fiz. **57**, 590 (1994) [Phys. At. Nucl. **57**, 556 (1994)].
[61] C. Ardisson, V. Berci, J. Dolmasso *et al.*, Nuovo Cimento A **105**, 215 (1992).
[62] F. K. McGowan, W. T. Milner, A. R. Robinson, and P. H. Stelson, Nucl. Phys. **A297**, 51 (1978).
[63] J. D. Panar and D. G. Burke, Can. J. Phys. **57**, 1999 (1979).
[64] D. G. Burke and P. E. Garret, Nucl. Phys. **A550**, 21 (1992).
[65] W. F. Davidson, D. D. Warner, R. F. Casten *et al.*, J. Phys. G **7**, 455 (1981).
[66] D. G. Burke, B. L. Maddock, and W. F. Davidson, Nucl. Phys. **A442**, 424 (1985).
[67] D. G. Burke, W. F. Davidson, J. A. Cizewski *et al.*, Nucl. Phys. **A445**, 70 (1985).
[68] D. G. Burke, W. F. Davidson, J. A. Cizewski *et al.*, Can. J. Phys. **63**, 1309 (1985).
[69] I. M. Govil, H. W. Filbright, D. Cline *et al.*, Phys. Rev. C **33**, 793 (1986).
[70] W. F. Davidson and W. R. Dixon, J. Phys. G **17**, 1683 (1991).
[71] P. Petkov, W. Andrejtscheff, J. Copnell, and S. J. Robinson, Nucl. Phys. **A533**, 49 (1991).
[72] H. G. Börner, J. Jolie, S. J. Robinson *et al.*, Phys. Rev. Lett. **66**, 691 (1991).
[73] R. Neu and F. Hoyler, Phys. Rev. C **46**, 208 (1992).
[74] A. Aprahamian, Phys. Rev. C **46**, 2093 (1992).
[75] A. Jungclaus, R. F. Casten, R. L. Gill *et al.*, Phys. Rev. C **49**, 88 (1994).
[76] K. M. Zheleznova, A. A. Korneĭchuk, V. G. Solov'ev *et al.*, Report D-2157, JINR, Dubna (1965) [in Russian].
[77] V. G. Solov'ev, At. Energ. Rev. **3**, No. 2, 117 (1965).
[78] V. O. Nesterenko, V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, Yad. Fiz. **44**, 1443 (1986) [Sov. J. Nucl. Phys. **44**, 938 (1986)].
[79] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, to be published.
[80] V. G. Solov'ev and A. V. Sushkov, J. Phys. G **16**, L57 (1990).
[81] V. G. Solov'ev, Z. Phys. A **334**, 143 (1989).
[82] A. A. Shihab-Eldin, J. O. Rasmussen, M. A. Stoyer *et al.*, Int. J. Mod. Phys. E **4**, 411 (1995).
[83] R. F. Casten and P. von Brentano, Phys. Rev. C **50**, R1280 (1994).
[84] R. F. Casten and P. von Brentano, Phys. Rev. C **51**, 3528 (1995).
[85] D. G. Burke and P. C. Sood, Phys. Rev. C **51**, 3525 (1995).
[86] W. T. Chou, R. F. Casten, and P. von Brentano, Phys. Rev. C **45**, R9 (1992).
[87] R. F. Casten, P. von Brentano, and N. V. Zampfir, Phys. Rev. C **49**, 1940 (1994).
[88] P. C. Sood, D. M. Headly, and R. K. Sheline, At. Data Nucl. Data Tables **47**, 89 (1991).
[89] D. G. Burke, in *Frontiers in Nuclear Physics*, Report D4-95-308, JINR, Dubna (1995), p. 41.
[90] V. G. Solov'ev, A. V. Sushkov, N. Yu. Shirikova, and N. Lo Iudice, Nucl. Phys. **A600**, 155 (1996).
[91] V. G. Solov'ev, Phys. Rev. C **51**, R2885 (1995).
[92] V. G. Solov'ev, A. V. Sushkov, and N. Yu. Shirikova, J. Phys. G **21**, 1217 (1995).
[93] V. G. Solov'ev and N. Yu. Shirikova, Z. Phys. A **334**, 149 (1989).
[94] A. F. Barfield, B. R. Barrett, J. L. Wood, and O. Scholten, Ann. Phys. (N.Y.) **182**, 344 (1988).
[95] M. Oshima, T. Morikawa, H. Kusakari *et al.*, Nucl. Phys. **A557**, 635c (1993).
[96] C. Fahlander, A. Axelsson, M. Heinebrodt *et al.*, Phys. Lett. **B**, (1996) [in press].
[97] D. F. Winchell, I. Y. Lee, C. Baktash *et al.*, Phys. Rev. C **51**, 2952 (1995).
[98] X. Wu, A. Aprahamian, J. Castro-Ceron, and C. Baktash, Phys. Lett. **316B**, 235 (1993).
[99] X. Wu, A. Aprahamian, S. M. Fischer *et al.*, Phys. Rev. C **49**, 1837 (1994).
[100] D. G. Burke, Phys. Rev. Lett. **73**, 1899 (1994).

Translated by Patricia A. Millard